

# MATHCOUNTS®

2014  
 ■ School Competition ■  
 Sprint Round  
 Problems 1–30

Name \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. 17 candy bars At a candy store Alexis purchased 3 candy bars for \$1.50. At this rate, how many whole candy bars can she buy with \$8.75?  

$$\begin{array}{r} 30.50 \text{ each} \\ 3 \overline{) 11.50} \\ \underline{9} \phantom{00} \\ 25 \phantom{00} \\ \underline{21} \phantom{00} \\ 40 \phantom{00} \\ \underline{30} \phantom{00} \\ 10 \phantom{00} \\ \underline{9} \phantom{00} \\ 10 \phantom{00} \\ \underline{9} \phantom{00} \\ 10 \phantom{00} \\ \underline{9} \phantom{00} \\ 10 \phantom{00} \end{array}$$
2. 25 If  $x \boxtimes y = (x + y)^2$  for positive integers  $x$  and  $y$ , then  $4 \boxtimes 5 = (4 + 5)^2 = 9^2 = 81$ . What is the value of  $2 \boxtimes 3$ ?  
 $(2+3)^2 = (5)^2 = 25$
3. 65 °F Consider the weather data for 10 days shown in the table below. John identified the mode of the daily high temperature and the mode of the daily low temperature, in degrees Fahrenheit, based on this data. What is the mean of those two numbers?  

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.
High	67	74	70	69	72	74	78	79	72	70
Low	61	68	60	65	63	65	63	58	60	59

$$\frac{72 + 58}{2} = \frac{130}{2} = 65$$
4. 2675 Karina wrote the addition problem shown here on the board. However, one digit is incorrect. When written correctly, the number containing the incorrect digit is what four-digit integer?  

$$\begin{array}{r} 2675 \\ + 1862 \\ \hline 4537 \end{array}$$
5. 4 triangles Point A is a vertex of a regular hexagon. When all possible diagonals from point A are drawn in this hexagon, how many triangles are formed?
6. 7 The function  $y = 3x + 6$  is graphed on a coordinate plane. The  $y$ -coordinate of a point on the line is 27. What is the  $x$ -coordinate of that point?  

$$\begin{array}{r} y = 3x + 6 \\ (27) = 3x + 6 \\ + -6 = + -6 \end{array}$$

$$\frac{21}{3} = \frac{3x}{3}$$

$$7 = x$$

7. 100 yards Two men stand back-to-back and walk in opposite directions for 40 yards each. Each of them then turns left and walks another 30 yards each. In yards, how far are the two men from one another?

8. 4 Jacinta rolled a standard six-sided die repeatedly until she got every number from 1 to 6. She stopped rolling the die when she'd gotten each number at least once. The line plot shows how many times each number occurred. What number occurred on her last roll?

Number of Times 1 through 6 Rolled
x
x
x x x
x x x
x x x x x x
x x x x x x
1 2 3 4 5 6

9. 16,200 pounds If 1 bag of seed covers exactly 90 square meters, and each square meter produces 60 pounds of oats, how many pounds of oats will be produced if 3 bags are planted?

$$3 \text{ bags} = 3 \times 90 = 270 \text{ m}^2$$

$$\begin{array}{r} 270 \text{ m}^2 \\ \times 60 \text{ lbs} \\ \hline 16,200 \end{array}$$

10. 5 What is the sum of the two integers between which  $\frac{\sqrt{17}}{2}$  lies?

$$\sqrt{16} = 4 \quad \frac{\sqrt{17}}{2} \approx 2.5 \quad \sqrt{25} = 5$$

11. 11 fractions How many common fractions strictly between 0 and 1 have a denominator of at most 6 when written in lowest terms?

$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$

12. 5 What is the sum of the exponents when  $\frac{(x^4y^3)(x^2y)^3}{(x^3y^2)(x^2y)}$  is written in simplest form with all exponents non-negative?

$$\frac{(x^4y^3)(x^2y)^3}{(x^3y^2)(x^2y)} = \frac{x^4y^3x^6y^3}{x^5y^3} = x^{4+6-5}y^{3+3-3} = x^5y^3$$

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13.  $\frac{13}{6}$  What is the absolute difference between the additive inverse and multiplicative inverse of  $\frac{2}{3}$ ? Express your answer as a common fraction.

$$-\frac{2}{3} \times 2 + \frac{-3 \times 3}{2 \times 3} \Rightarrow -\frac{4}{3} + \frac{-9}{6} = -\frac{13}{6}$$

14.  $192\frac{1}{3}$  Three integers greater than 190 but less than 200 have a sum of 577. What is the average of the three numbers? Express your answer as a mixed number.

$$3 \overline{) 577} = 192\frac{1}{3}$$

15. 0.3 A jar is filled with red, orange and yellow jelly beans. The probability of randomly selecting a red jelly bean from this jar is 0.2, and the probability of randomly selecting an orange jelly bean from this jar is 0.5. What is the probability of randomly selecting a yellow jelly bean from this jar? Express your answer as a decimal to the nearest tenth.

red = 20%  
orange = 50%  
yellow = leftover  
Total = 100%  
 $100 - 20 - 50 = 30\%$

16. \$ 60 One Saturday, Sam and Joe worked together to mow yards. Sam started at 8:00 a.m. and Joe started at 11:00 a.m. When they finished at 4:00 p.m., they had earned a total of \$156. They want to share the money they earned fairly based on the number of hours worked. How much should Joe receive?

Sam = 8 hours  
Joe = 5 hours  
Total = 13 hours

$$\frac{5 \times 12}{13 \times 12} = \frac{5}{13} \rightarrow J = \$60$$

17. (4, 3) At what point do the lines  $y = 2x - 5$  and  $y = -\frac{1}{2}x + 5$  intersect? Express your answer as an ordered pair.

$$2x - 5 = -\frac{1}{2}x + 5$$

$$\frac{3}{2}x = 10 \rightarrow x = \frac{20}{3} = 6\frac{2}{3}$$

$$y = 2(6\frac{2}{3}) - 5 = 13\frac{4}{3} - 5 = 8\frac{1}{3}$$

18. 21 The positive integer  $n$  is 100 less than one perfect square and 28 less than another perfect square. If  $n < 50$ , what is the value of  $n$ ?

$$\begin{array}{r} 72 \\ + 5 \\ \hline 77 \\ \hline 121 \\ \hline 144 \end{array}$$


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19. 2/3 In one middle school, 300 students play sports and 200 students are on the Honor Roll. If 120 of the students on the Honor Roll also play sports, what is the probability that a randomly chosen student who plays a sport is also on the Honor Roll? Express your answer as a common fraction.

$$\frac{120 \div 6}{300 \div 6} = \frac{2}{3}$$

20. 3 dimes Tai Lynn has 12 common US coins that add to 83¢. She has exactly five coins of one type. How many dimes does she have?


$$\begin{array}{r} 76 \\ + 3 \\ \hline 79 \end{array}$$

21. 10 drops  A veterinary assistant needs to prepare a mixture of a vitamin solution for parakeets. The ratio is 1 drop of vitamin oil to 25 drops of water. If the assistant needs to prepare 260 drops of the combined solution, how many drops of the vitamin oil will she need?

$$1 + 25 = 26 \overline{)260} \quad 10 \text{ groups}$$

22. 5 ways How many ways can the integers 1 through 6 be arranged in the 2-by-3 grid shown such that each row and each column have their numbers in increasing order?

1	2	3	4	5	6
3	5	1	2	3	2

23. 93 games  A baseball team has won 15 games and lost 9. If these 24 games represent  $\frac{1}{6}$  of the games played during the entire season, how many more games must the team win in order to win  $\frac{3}{4}$  of their games for the season?

$$\frac{108}{6} \times 24 = 144$$

$$\frac{3 \times 144}{4} = 108$$

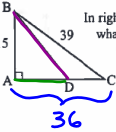
$$144 - 108 = 36$$

24. An arithmetic progression is formed by five distinct prime numbers. What is the least possible sum of those five numbers?

7, 11, 13, 17, 19, 23, 29

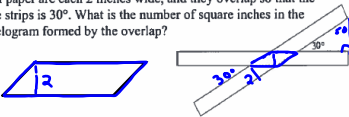
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25. 20 units In right triangle ABC, BD = CD + 9. If AB = 15 and BC = 39, what is AD?



see notes

26. in<sup>2</sup> Two long strips of paper are each 2 inches wide, and they overlap so that the angle between the strips is 30°. What is the number of square inches in the area of the parallelogram formed by the overlap?



h=2  
b=?

27. 200,000 people Every year, 20% of a city's population moves to the suburbs and 10% of the suburban population moves to the city. Remarkably, this migration does not change their respective populations. If the city and suburbs have a total population of 600,000 people, how many people live in the city?

$$C + b = 600,000$$

$$0.2c = 0.1b$$

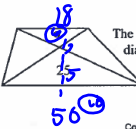
see notes

28. 55 coins Amy has \$5 in quarters, dimes and nickels, where the total value in quarters is more than the total value of the dimes, and the total value of the dimes is more than the total value of the nickels. What is the maximum number of coins Amy could have?

$$3.35 \quad 1.75 \quad 1.70 \quad 1.55$$

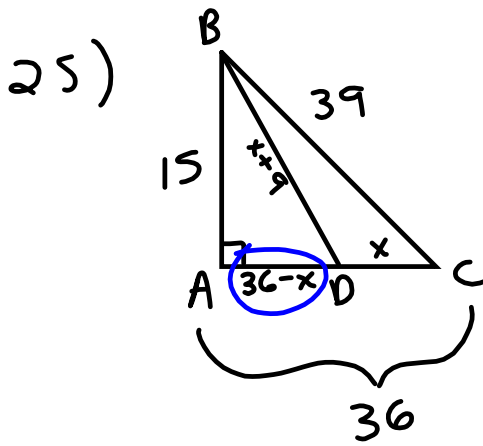
$$7 + 17 + 31 = 55$$

29. The four vertices of a square are (-5, 4), (1, 4), (1, -2) and (-5, -2). What is the slope of the line that passes through the origin and divides the square into two congruent trapezoids? Express your answer as a common fraction.

30. 64 units<sup>2</sup>  The trapezoid shown is divided into four triangles by its diagonals. The areas of two triangles are indicated. What is the area of the whole trapezoid?

$$16 \times 4 = 64$$

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$$15^2 + (36-x)^2 = (x+9)^2$$

$$225 + 36^2 - 72x + x^2 = x^2 + 18x + 81$$

$$225 + 1296 - 72x + x^2 = x^2 + 18x + 81$$

$$\begin{array}{r} 1296 \\ 225 \\ \hline 1521 \end{array} - 72x + x^2 = x^2 + 18x + 81$$

$$\begin{array}{r} -81 \\ \hline 1440 \end{array} + 72x = +72x - 81$$

$$1440 = 90x$$

$$x = 16$$

$$9 \overline{) 144} \begin{array}{r} 16 \\ -9 \\ \hline 54 \end{array}$$

27)

$$c + b = 600,000$$

$$\frac{0.2c}{0.1} = \frac{0.1b}{0.1}$$

$$2c = b$$

$$b = 2c$$

$$c + (2c) = 600,000$$

$$\frac{3c}{3} = \frac{600,000}{3}$$

$$c = 200,000$$