9-1

NAME _____ DATE _____ PERIOD _____

Study Guide

Student Edition Pages 382–387

Polynomials

The expressions y, $-6x$, $5a^2$, and $10cd^3$ are examples of **monomials**. A monomial is a number, a variable, or a product of numbers and variables. Any exponents in a monomial are positive integers. The exponents cannot be variables.

Example: Which of the following expressions are monomials?

$-8g$ $\quad 2z^{-4}$ $\quad 17t^5v$ $\quad 3 - a$ $\quad \frac{9}{pq}$ $\quad 6^y$

$-8g$ and $17t^5v$ are monomials because they are products of numbers and variables.

$2z^{-4}$ is not a monomial because it has a negative exponent.
$3 - a$ is not a monomial because it involves subtraction.
$\frac{9}{pq}$ is not a monomial because it involves division.
6^y is not a monomial because its exponent is a variable.

The sum of two or more monomials is called a **polynomial**. Each monomial is a term of the polynomial. Polynomials with two or three terms have special names.

$15r^4 + 1$ is a **binomial**. It has two terms, $15r^4$ and 1.

$-9 + g - 4g^2$ is a **trinomial**. It has three terms, -9, g, and $-4g^2$.

Determine whether each expression is a monomial. Explain why or why not.

1. $18x + 2$ Not monomial It's a binomial (2 terms)
2. $-21s^4t^2$ monomial product of number and variables; also + exponents
3. w^{-2} not a monomial due to negative exponent
4. $\frac{4}{5}a^3b$ not a monomial due to division

State whether each expression is a polynomial. If it is a polynomial, identify it as a monomial, binomial, *or* trinomial.

5. $\frac{8}{x}$ not polynomial (÷)
6. $-7r + 9s + 3$ trinomial
7. $abc^3 - a^3bc$
8. $35u^5v^6$
9. $5 + 8^{\circ}$ not polynomial
10. $8d - 9e \div f$ not polynomial (÷)
11. $16x - 16y$
12. $8j^2 + 3j - 7$
13. $3m^3 + \frac{1}{3}m$ not polynomial (÷)
14. $-14p + p^{-14}$ not polynomial (− exp.)

9-1

NAME _____ DATE _____ PERIOD _____

Practice

Student Edition Pages 382–387

Polynomials

Determine whether each expression is a monomial. Explain why or why not.

1. $8y^2$
2. $3m^{-4}$
3. $\frac{6}{p}$
4. -9
5. $2x^2 + 5$
6. $-7a^3b$

State whether each expression is a polynomial. If it is a polynomial, identify it as a monomial, binomial, *or* trinomial.

7. $4h + 8$
8. 13
9. $3xy$
10. $\frac{2}{c} + 4$
11. $m^2 + 2 - m$
12. $5a + b^{-2}$
13. $7 - \frac{1}{2}d$
14. n^2
15. $2a^2 + 8a + 9 - 3$
16. $x^3 + 4x^3$
17. $m^2 + 2mn + n^2$
18. $6 - y$

Find the degree of each polynomial.

19. $8 \rightarrow 0$
20. $3a^2 \rightarrow 2$
21. $5m^1 + n^2 \rightarrow 2$
22. $16cd$
23. $3g^4 + 2h^3 \rightarrow 4$
24. $4a^2b^1 + 3ab^3 \rightarrow 4$
25. $c^2 + 2c - 8$
26. $2p^3 - 7p^2 - 4p$
27. $9y^3z^1 + 15y^5z^1 \rightarrow 6$
28. $7s^2 - 4s^2t + 2st$
29. $6x^3 + x^3y^2 - 3$
30. $2ab^3 - 5abc$

9-2

NAME _____ DATE _____ PERIOD _____

Study Guide

Student Edition Pages 388–393

Adding and Subtracting Polynomials

To add polynomials, group the like terms together and then find the sum. $3x^2$ and x^2 are like terms. x^2 and x, and x^2 and y^2 are unlike terms.

Example 1: Find $(3x^2 + 2x - 5) + (x^2 + 4x + 4)$.

$(3x^2 + 2x - 5) + (x^2 + 4x + 4)$
$= (3x^2 + 2x + (-5)) + (x^2 + 4x + 4)$ *Rewrite subtraction.*
$= (3x^2 + x^2) + (2x + 4x) + (-5 + 4)$ *Regroup like terms.*
$= (3 + 1)x^2 + (2 + 4)x + (-5 + 4)$ *Distributive property*
$= 4x^2 + 6x - 1$ *Simplify.*

You can subtract a polynomial by adding its additive inverse.

Example 2: Find the additive inverse of $5b^2 - 3$.

The additive inverse is $-(5b^2 - 3)$ or $-5b^2 + 3$.

Example 3: Find $(4m^3 - 6) - (7m^3 - 9)$.

$(4m^3 - 6) - (7m^3 - 9)$
$= (4m^3 - 6) + (-7m^3 + 9)$ *The additive inverse of $7m^3 - 9$ is $-7m^3 + 9$.*
$= (4m^3 - 7m^3) + (-6 + 9)$ *Regroup like terms.*
$= (4 - 7)m^3 + (-6 + 9)$ *Distributive property*
$= -3m^3 + 3$ *Simplify.*

Find each sum or difference.

1. $(2a + 3) + (5a + 1)$
2. $(8w^2 + w) + (7w^2 - 3w)$
3. $(-5c^4 + 2c^2 - 6) + (6c^4 - 2c^2 + 5)$
4. $(12m - 5n) + (12m + 5n)$
5. $(4g + h^3) + (-9g - 4h^3)$
6. $(2 - 16x^2) + (8 - 16x^2)$
7. $(18 + 5xy) + (-6 - 10xy)$
8. $(35a^2 + 15a + 20) + (10a^2 + 25)$
9. $(6d + 3) - (4d + 5)$
10. $(14 - 3t) - (2 + 7t)$
11. $(-18s^2 + s) - (6s^2 - 8s)$
12. $(26g - 13gh) - (-2g + gh)$
13. $(7y^2 + 2y + 21) + (9y^2 - 6y - 11)$
14. $(-5m^2 + 2n - 1) - (7m^2 + 16n - 8)$

9-2

NAME _____ DATE _____ PERIOD _____

Practice

Student Edition Pages 388–393

Adding and Subtracting Polynomials

Find each sum.

1. $5x - 2$ / $(+)\ 4x + 6$
2. $2y + 4$ / $(+)\ y - 1$
3. $4x - 8$ / $(+)\ 2x + 5$
4. $2x^2 + 7x - 4$ / $(+)\ x^2 + 3x + 2$
5. $n^2 + 4n + 3$ / $(+)\ 3n^2 + 4n - 4$
6. $2x^2 + 3xy - y^2$ / $(+)\ 2x^2 - 2xy - 4y^2$
7. $(2x^2 - 2x - 4) + (x^2 - 3x + 2)$
8. $(x^2 + 2x - 1) + (3x^2 - 4x + 1)$
9. $(2a^2 + 8a + 6) + (a^2 + 3a - 4)$
10. $(x^2 + x - 12) + (x^2 - 3x)$
11. $(3x^2 + 8x + 4) + (4x^2 - 1)$
12. $(x^2 - 4x - 5) + (x^2 + 4x)$

Find each difference.

13. $7n + 2$ / $(-)\ n + 1$
14. $3x - 3$ / $(-)\ 2x + 2$
15. $2y + 5$ / $(-)\ y - 1$
16. $4x^2 + 7x - 2$ / $(-)\ 2x^2 + 6x + 4$
17. $2x^2 + 9x + 5$ / $(-)\ x^2 + 5x + 6$
18. $5m^2 - 4m - 1$ / $(-)\ 4m^2 + 8m + 4$
19. $(6x - 2) - (8x + 3)$
20. $(3x^2 - 3x - 6) - (2x^2 - 2x - 4)$
21. $(6x^2 + 2x - 8) - (4x^2 + 8x + 4)$
22. $(2a^2 + 6a + 4) - (a^2 - 3)$
23. $(2x^2 - 8x + 3) - (-x^2 + 2x)$
24. $(3x^2 - 5xy - 2y^2) - (2x^2 + y^2)$

9-3

NAME _____ DATE _____ PERIOD _____

Study Guide

Student Edition Pages 394–398

Multiplying a Polynomial by a Monomial

You can use the Distributive Property to multiply a polynomial by a monomial.

Example 1: Multiply $6(x + 7)$.

$6(x + 7) = 6(x) + 6(7)$ *Distributive Property.*
$= 6x + 42$ *Simplify.*

Example 2: Multiply $-2(g^2 + 3g - 5)$.

$-2(g^2 + 3g - 5) = -2(g^2) + (-2)(3g) + (-2)(-5)$
$= -2g^2 - 6g + 10$

Example 3: Multiply $9a(a + 1)$.

$9a(a + 1) = 9a(a) + 9a(1)$
$= 9a^2 + 9a$

Some equations require that you multiply polynomials.

Example 4: Solve $5(x + 3) = 25$.

$5(x + 3) = 25$
$5x + 15 = 25$ *Distributive Property*
$5x + 15 - 15 = 25 - 15$ *Subtract 15 from each side.*
$5x = 10$ *Combine like terms.*
$x = 2$ *Divide each side by 5.*

Find each product.

1. $8(x + 2)$ → $8x + 6$
2. $-3(x + 5)$ → $-3x + -15$
3. $10(2a - b)$
4. $4(v^2 - 4v + 9)$
5. $-3y(y - 1)$
6. $2r(-2r^2 + 6r + 5)$ → $-4r^3 + +12r^2 + -10r$
7. $0.3(2p + 4)$
8. $4.5(m^3 + m^2)$
9. $\frac{1}{2}(z + 10)$

Solve each equation.

10. $2(y + 3) = 10$
11. $-5(x + 8) = 5$
12. $7(3s - 1) = -49$
13. $-4(-2w + 7) = -20$
14. $6(d + 1) - 6 = 18$

9-3

NAME _____ DATE _____ PERIOD _____

Practice

Student Edition Pages 394–398

Multiplying a Polynomial by a Monomial

Find each product.

1. $3(y + 4)$
2. $-2(n + 3)$
3. $5(3a - 4)$
4. $7(-2c + 3)$
5. $x(x + 6)$
6. $8y(2y - 3)$
7. $y(9 + 2y)$
8. $-3b(b + 1)$ → $(-3b)(b) + (-3b)(-1)$ → $-3b^2 + +3b$
9. $6(a^2 + 5)$
10. $-4m(-2 + 2m)$
11. $-7n(-4n + 2)$
12. $2q(3q - 1)$
13. $p(3p^2 + 7)$
14. $4x(5 - 2x^2)$
15. $5b(b^2 + 5b)$
16. $-3y(-9 + 3y^2)$
17. $2(8a^2 - 4a + 9)$
18. $6(z^2 + 2z - 6)$
19. $x(x^2 - x + 3)$
20. $-4b(1 - 7b + b^2)$
21. $5m^2(3m^2 - m - 7)$
22. $-7y(-2 + 7y + 3y^2)$
23. $-3n^2(n^2 + 2n + 3)$ → $(-3n^2)(n^2) + (-3n^2)(-2n) + (-3n^2)(3)$ → $-3n^4 + +6n^3 + -9n^2$
24. $9c(2c^3 + c^2 - 4)$

Solve each equation.

25. $5(y + 2) = 25$
26. $7(x - 2) = -7$
27. $2(a - 5) + 4 = a + 9$
28. $3(2x + 6) - 10 = 4(x + 3)$ see notes
29. $-6(2n - 2) + 12 = 4(2n - 9)$
30. $b(b + 8) = b(b + 7) + 5$
31. $y(y + 7) + 3y = y(y + 3) - 14$
32. $m(m - 5) + 14 = m(m + 2) - 14$ see notes

28) $3(2x+6)+10 = 4(x+3)$ +20

+20

$6x + 18 + -10 = 4x + 12$

$6x + +8 = 4x + 12$

$+-8 \quad = \quad +-8$

$6x = 4x + +4$

$+-4x \quad = +-4x$

$2x = 4$

$2 \quad 2$

$x = 2$ ✓

32) $m(m+-5)+14 = m(m+2)+-14$ +10

+10

$m^2 + -5m + +14 = m^2 + 2m + -14$

$+-14 = \quad +-14$

$m^2 + -5m = m^2 + -28$

$+-2m \quad = \quad +-2m$

$m^2 + -7m = m^2 + -28$

$+-m^2 \quad = -m^2$

$-7m = -28$

$-7 \quad -7$

$m = +4$ ✓

9-4

NAME _____ DATE _____ PERIOD _____

Study Guide

Student Edition
Pages 399–404

Multiplying Binomials

The Distributive Property can be used to multiply binomials.

Example 1: Multiply $(x + 2)(x + 5)$.

$(x + 2)(x + 5) = x(x + 5) + 2(x + 5)$ *Distributive Property*
$= x(x) + x(5) + 2(x) + 2(5)$ *Distributive Property*
$= x^2 + 5x + 2x + 10$ *Simplify.*
$= x^2 + 7x + 10$ *Combine like terms.*

Example 2: Multiply $(3x + 1)(x - 4)$.

$(3x + 1)(x - 4) = 3x(x - 4) + 1(x - 4)$ *Distributive Property*
$= 3x(x) + 3x(-4) + 1(x) + 1(-4)$ *Distributive Property*
$= 3x^2 - 12x + x - 4$ *Simplify.*
$= 3x^2 - 11x - 4$ *Combine like terms.*

You can also use a shortcut called the **FOIL method** to multiply two binomials. Find the four products indicated by the letters in the word FOIL. Then add the like terms.

Example 3: Multiply $(y + 4)(y - 3)$.

F O I L
First terms + Outer terms + Inner terms + Last terms

$(y + 4)(y - 3) = y(y) + y(-3) + 4(y) + 4(-3)$
$= y^2 - 3y + 4y - 12$ *Add the like terms.*
$= y^2 + y - 12$

Find each product.

1. $(x + 6)(x + 3)$ see notes
2. $(y + 4)(y - 2)$
3. $(m - 3)(m - 1)$ see notes
4. $(h - 10)(h + 7)$
5. $(w - 8)(w - 8)$
6. $(g - 2)(g + 2)$
7. $(2a + 5)(a + 3)$
8. $(p + 1)(2p + 3)$
9. $(3x - 4)(x + 1)$
10. $(z + 5)(4z - 3)$ see notes

1) $(x+6)(x+3)$

$(x)(x) + (x)(3) + (6)(x) + (6)(3)$

$x^2 + 3x + 6x + 18$

$x^2 + 9x + 18$

3) $(m-3)(m-1)$

$(m)(m) + (m)(-1) + (-3)(m) + (-3)(-1)$

$m^2 + -1m + -3m + +3$

$m^2 + -4m + +3$

10) $(z+5)(4z-3)$

$(z)(4z) + (z)(-3) + (5)(4z) + (5)(-3)$

$4z^2 + -3z + 20z + -15$

$4z^2 + 17z + -15$

9-4 NAME _____ DATE _____ PERIOD _____

Practice

Student Edition Pages 399–404

Multiplying Binomials

Find each product. Use the Distributive Property or the FOIL method.

1. $(y + 4)(y + 3)$
2. $(x + 2)(x + 1)$
3. $(b + 5)(b - 2)$
4. $(a - 6)(a - 4)$
5. $(z - 5)(z + 3)$
6. $(n - 1)(n - 8)$
7. $(x + 7)(x - 4)$
8. $(y - 3)(y + 9)$
9. $(b + 2)(b + 3)$
10. $(2c + 5)(c - 4)$
11. $(4x - 7)(x + 3)$
12. $(x - 1)(5x - 4)$
13. $(3y + 1)(3y + 2)$
14. $(2n + 4)(5n - 3)$
15. $(7h - 3)(4h - 1)$
16. $(2m - 6)(3m + 2)$
17. $(6a + 2)(2a + 3)$
18. $(4c + 5)(2c - 2)$
19. $(x + y)(2x + y)$
20. $(3a + 4b)(a - 3b)$
21. $(3m - 3n)(3m - 2n)$ see notes
22. $(7p - 4q)(2p + 3q)$
23. $(2r + 2s)(2r + 3s)$
24. $(3y - 5z)(3y + 3z)$
25. $(x^2 + 1)(x - 3)$ see notes
26. $(y - 4)(y^2 + 2)$
27. $(2c^2 - 5)(c - 4)$
28. $(a^3 - 3a)(a + 4)$
29. $(b^2 + 2)(b^2 + 3)$ see notes
30. $(x^3 - 3)(4x + 1)$

21) $(3m - 3n)(3m - 2n)$

$(3m)(3m) + (3m)(-2n) + (-3n)(3m) + (-3n)(-2n)$

$9m^2 + -6mn + -9nm + +6n^2$

$-9mn$

$9m^2 + -15mn + 6n^2$

25) $(x^2+1)(x+3)$

$(x^2)(x) + (x^2)(3) + (1)(x) + (1)(3)$

$x^3 + 3x^2 + x + 3$

29) $(b^2+2)(b^2+3)$

$b^2(b^2) + b^2(3) + 2(b^2) + 2(3)$

$b^4 + 3b^2 + 2b^2 + 6$

$b^4 + 5b^2 + 6$

9-5

NAME _____ DATE _____ PERIOD _____

Study Guide

Student Edition Pages 405–409

Special Products

Recall x^2 means that x is used as a factor twice. Thus, $x^2 = x \cdot x$. When a binomial is squared, it is also used as a factor twice.

Therefore $(x + 3)^2 = (x + 3)(x + 3)$.
$= x^2 + 3x + 3x + 9$ *The inner and outer products are equal.*
$= x^2 + 6x + 9$

Similarly, $(x - 3)^2 = (x - 3)(x - 3)$.
$= x^2 - 3x - 3x + 9$ *The inner and outer products are equal.*
$= x^2 - 6x + 9$

Look at this special product of two binomials.
$(x + 3)(x - 3) = x^2 + 3x - 3x - 9$ *The inner and outer products are opposites.*
$= x^2 - 9$

Square of a Sum	Square of a Difference	Product of a Sum and a Difference
$(a + b)^2 = a^2 + 2ab + b^2$	$(a - b)^2 = a^2 - 2ab + b^2$	$(a + b)(a - b) = a^2 - b^2$

Example 1: Find $(r - 7)^2$.

$(a - b)^2 = a^2 - 2ab + b^2$ *Square of a difference*
$(r - 7)^2 = r^2 - 2(r)(7) + 7^2$ *Replace a with r and b with 7.*
$= r^2 - 14r + 49$

Example 2: Find $(6y - 5)(6y + 5)$.

$(a + b)(a - b) = a^2 - b^2$ *Product of a sum and a difference*
$(6y - 5)(6y + 5) = (6y)^2 - 5^2$ *Replace a with 6y and b with 5.*
$= 36y^2 - 25$

Find each product.

1. $(y + 8)^2$ see notes
2. $(z - 4)^2$ $z^2 - 8z + 16$
3. $(9 + a)^2$
4. $(5b + 1)^2$ see notes
5. $(3d - e)^2$ $9d^2 - 6de + e^2$
6. $(1 + 5j)^2$
7. $(x + 5)(x - 5)$ $x^2 - 5x + 5x - 25$ $x^2 - 25$
8. $(q + 7)(q - 7)$
9. $(m + 10)(m - 10)$
10. $(k + 2)(k - 2)$ $k^2 - 4$
11. $(6x + 1)(6x - 1)$ $36x^2 - 1$
12. $(2s + 3)(2s - 3)$

1) $(1y + 8)^2 \rightarrow (y + 8)(y + 8)$

$(y)(y) + (y)(8) + (8)(y) + (8)(8)$

$y^2 + 8y + 8y + 64$

$1y^2 + 16y + 64$

1st term squared $\rightarrow (y)^2$

Last term squared $\rightarrow (8)^2$

multiply $(y \cdot 8)$ then double

$\hookrightarrow (y \cdot 8)2 = 16y$

4) $(5b+1)^2 \rightarrow (5b+1)(5b+1)$

$(5b)(5b)+(5b)(1)+(1)(5b)+(1)(1)$

$25b^2 + 5b + 5b + 1$

$25b^2 + 10b + 1$

1st term

$(5b)^2 \rightarrow 25b^2$

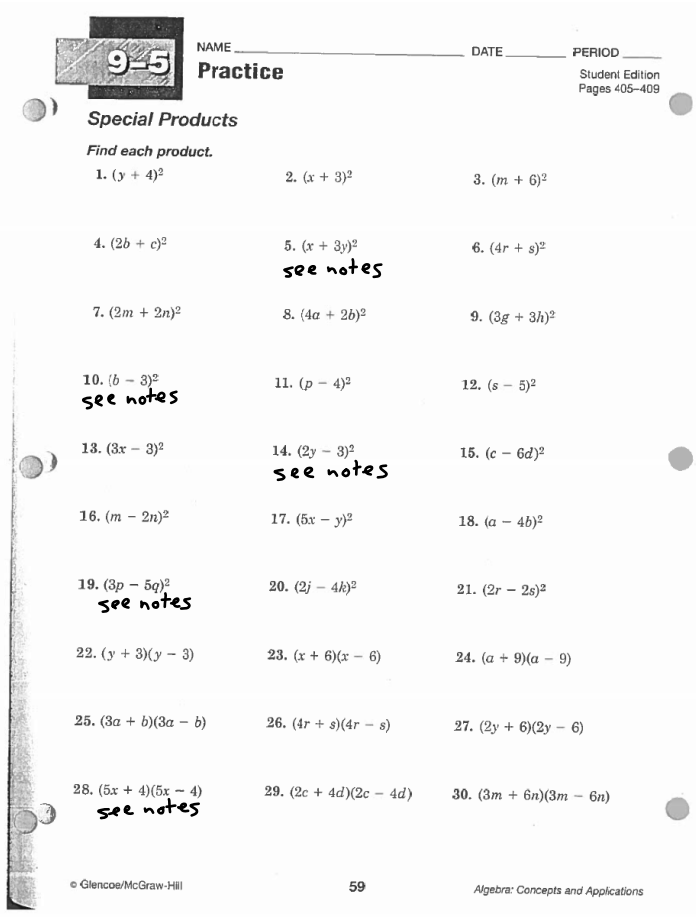
$(5b)(5b)$

Last term $\rightarrow (1)^2 \rightarrow 1$

middle term

$(5b)(1)(2) \rightarrow 10b$

$25b^2 + 10b + 1$



9–5 Practice

NAME _____ DATE _____ PERIOD _____

Student Edition Pages 405–409

Special Products

Find each product.

1. $(y + 4)^2$
2. $(x + 3)^2$
3. $(m + 6)^2$
4. $(2b + c)^2$
5. $(x + 3y)^2$ see notes
6. $(4r + s)^2$
7. $(2m + 2n)^2$
8. $(4a + 2b)^2$
9. $(3g + 3h)^2$
10. $(b - 3)^2$ see notes
11. $(p - 4)^2$
12. $(s - 5)^2$
13. $(3x - 3)^2$
14. $(2y - 3)^2$ see notes
15. $(c - 6d)^2$
16. $(m - 2n)^2$
17. $(5x - y)^2$
18. $(a - 4b)^2$
19. $(3p - 5q)^2$ see notes
20. $(2j - 4k)^2$
21. $(2r - 2s)^2$
22. $(y + 3)(y - 3)$
23. $(x + 6)(x - 6)$
24. $(a + 9)(a - 9)$
25. $(3a + b)(3a - b)$
26. $(4r + s)(4r - s)$
27. $(2y + 6)(2y - 6)$
28. $(5x + 4)(5x - 4)$ see notes
29. $(2c + 4d)(2c - 4d)$
30. $(3m + 6n)(3m - 6n)$

5) $(x+3y)^2$

$x^2+6yx+9y^2$

$3y \cdot x \cdot 2$

$3yx \cdot 2 = 6yx$

10) $(b-3)^2 \rightarrow (b-3)(b-3)$

$b^2 + -6b + 9$

$(b)(b) + (b)(-3) + (-3)(b) + (-3)(-3)$

$b^2 + -3b + -3b + +9$

$b^2 + -6b + 9$

14) $(2y-3)^2 \rightarrow (2y-3)(2y-3)$

$(2y)(2y) + (2y)(-3) + (-3)(2y) + (-3)(-3)$

$4y^2 + -6y + -6y + 9$

$4y^2 + -12y + 9$

$(2y)^2$ $2(2y)(-3)$ $(-3)^2$

$(2y)(2y)$ $-12y$ $(-3)(-3)$

$4y^2 +$ $+9$

19) $(3p+-5q)^2 \rightarrow (3p+-5q)(3p+-5q)$

$9p^2 + 30pq + 25q^2$

28) $(5x+4)(5x-4)$

$25x^2 + -20x + 20x + -16$

$25x^2 + -16$