Name: ____________________ Squares and Square Roots Date: __________

n	Squared^2 n^2	Square Root $\sqrt{n}$	n	Squared^2 n^2	Square Root $\sqrt{n}$	n	Square Root $\sqrt{n}$	n	Square Root $\sqrt{n}$	n	Square Root $\sqrt{n}$
1	1	1.000	51	2601	7.141	101	10.050	151	12.288	201	14.177
2	4	1.414	52	2704	7.211	102	10.100	152	12.329	202	14.213
3	9	1.732	53	2809	7.280	103	10.149	153	12.369	203	14.248
4	16	2.000	54	2916	7.348	104	10.198	154	12.410	204	14.283
5	25	2.236	55	3025	7.416	105	10.247	155	12.450	205	14.318
6	36	2.449	56	3136	7.483	106	10.296	156	12.490	206	14.353
7	49	2.646	57	3249	7.550	107	10.344	157	12.530	207	14.387
8	64	2.828	58	3364	7.616	108	10.392	158	12.570	208	14.422
9	81	3.000	59	3481	7.681	109	10.440	159	12.610	209	14.457
10	100	3.162	60	3600	7.746	110	10.488	160	12.649	210	14.491
11	121	3.317	61	3721	7.810	111	10.536	161	12.689	211	14.526
12	144	3.464	62	3844	7.874	112	10.583	162	12.728	212	14.560
13	169	3.606	63	3969	7.937	113	10.630	163	12.767	213	14.595
14	196	3.742	64	4096	8.000	114	10.677	164	12.806	214	14.629
15	225	3.873	65	4225	8.062	115	10.724	165	12.845	215	14.663
16	256	4.000	66	4356	8.124	116	10.770	166	12.884	216	14.697
17	289	4.123	67	4489	8.185	117	10.817	167	12.923	217	14.731
18	324	4.243	68	4624	8.246	118	10.863	168	12.961	218	14.765
19	361	4.359	69	4761	8.307	119	10.909	169	13.000	219	14.799
20	400	4.472	70	4900	8.367	120	10.954	170	13.038	220	14.832
21	441	4.583	71	5041	8.426	121	11.000	171	13.077	221	14.866
22	484	4.690	72	5184	8.485	122	11.045	172	13.115	222	14.900
23	529	4.796	73	5329	8.544	123	11.091	173	13.153	223	14.933
24	576	4.899	74	5476	8.602	124	11.136	174	13.191	224	14.967
25	625	5.000	75	5625	8.660	125	11.180	175	13.229	225	15.000
26	676	5.099	76	5776	8.718	126	11.225	176	13.266	226	15.033
27	729	5.196	77	5929	8.775	127	11.269	177	13.304	227	15.067
28	784	5.292	78	6084	8.832	128	11.314	178	13.342	228	15.100
29	841	5.385	79	6241	8.888	129	11.358	179	13.379	229	15.133
30	900	5.477	80	6400	8.944	130	11.402	180	13.416	230	15.166
31	961	5.568	81	6561	9.000	131	11.446	181	13.454	231	15.199
32	1024	5.657	82	6724	9.055	132	11.489	182	13.491	232	15.232
33	1089	5.745	83	6889	9.110	133	11.533	183	13.528	233	15.264
34	1156	5.831	84	7056	9.165	134	11.576	184	13.565	234	15.297
35	1225	5.916	85	7225	9.220	135	11.619	185	13.601	235	15.330
36	1296	6.000	86	7396	9.274	136	11.662	186	13.638	236	15.362
37	1369	6.083	87	7569	9.327	137	11.705	187	13.675	237	15.395
38	1444	6.164	88	7744	9.381	138	11.747	188	13.711	238	15.427
39	1521	6.245	89	7921	9.434	139	11.790	189	13.748	239	15.460
40	1600	6.325	90	8100	9.487	140	11.832	190	13.784	240	15.492
41	1681	6.403	91	8281	9.539	141	11.874	191	13.820	241	15.524
42	1764	6.481	92	8464	9.592	142	11.916	192	13.856	242	15.556
43	1849	6.557	93	8649	9.644	143	11.958	193	13.892	243	15.588
44	1936	6.633	94	8836	9.695	144	12.000	194	13.928	244	15.620
45	2025	6.708	95	9025	9.747	145	12.042	195	13.964	245	15.652
46	2116	6.782	96	9216	9.798	146	12.083	196	14.000	246	15.684
47	2209	6.856	97	9409	9.849	147	12.124	197	14.036	247	15.716
48	2304	6.928	98	9604	9.899	148	12.166	198	14.071	248	15.748
49	2401	7.000	99	9801	9.950	149	12.207	199	14.107	249	15.780
50	2500	7.071	100	10000	10.000	150	12.247	200	14.142	250	15.811

14-1 NAME ______________________ DATE ________ PERIOD ______

Study Guide

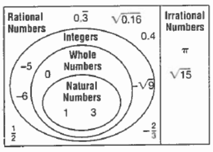
Student Edition
Pages 600–605

The Real Numbers

The numbers that we use every day belong to the set of real numbers. The real numbers can be divided into rational numbers and irrational numbers. Rational numbers can be expressed as fractions. Irrational numbers cannot be expressed as fractions. The set of rational numbers contains the natural numbers, the whole numbers, and the integers.

Natural Numbers: {1, 2, 3, 4, ...} → "counting"
Whole Numbers: {0, 1, 2, 3, ...}
Integers: {..., −2, −1, 0, 1, 2, ...}

The Venn diagram shows how subsets of the real numbers are related.



- 3 is a natural number, a whole number, an integer, and a rational number.
- 0 is a whole number, an integer, and a rational number.
- $-\sqrt{9}$ or 3 is an integer and a rational number.
- $\sqrt{15}$ or 3.872983346... is an irrational number.
- $0.\overline{3}$ or $\frac{1}{3}$ is a rational number.

Example: Find an approximation to the nearest tenth of $\sqrt{12}$.

On a graphing calculator, press $\boxed{\text{2nd}}$ [$\sqrt{\ }$] 12 $\boxed{\text{ENTER}}$.

The result is 3.464101615. So an approximate value for $\sqrt{12}$ is 3.5.

Name the set or sets of numbers to which each real number belongs. Let N = natural numbers, W = whole numbers, Z = integers, Q = rational numbers, and I = irrational numbers.

1. -4 Z, Q
2. $\frac{2}{5}$ Q
3. $-\sqrt{25}$ → −5 Z, Q
4. 10 N, W, Z, Q
5. 2.3 Q
6. $\sqrt{3}$ I
7. −4.324781... I
8. $-\frac{24}{8}$ Q
9. $\sqrt{100}$ Q
10. $\frac{1}{9}$ Q
11. −0.25 Q
12. $\sqrt{15}$ I

Find an approximation to the nearest tenth for each square root.

13. $\sqrt{2}$ ≈1.4
14. $\sqrt{14}$ ≈3.7
15. $-\sqrt{20}$ ≈ ⁻4.5
16. $\sqrt{55}$ ≈7.4

14–1 NAME ______________________ DATE ________ PERIOD ______

Practice

Student Edition Pages 600–605

The Real Numbers

Name the set or sets of numbers to which each real number belongs. Let N = natural numbers, W = whole numbers, Z = integers, Q = rational numbers, and I = irrational numbers.

1. $\sqrt{19}$ 2. -8 3. $1.737337\ldots$ 4. $0.\overline{4}$

5. $-\frac{5}{6}$ 6. $\sqrt{64}$ 7. $-\frac{28}{7}$ 8. $-\sqrt{144}$

9. $0.414114111\ldots$ 10. $\frac{1}{3}$ 11. 13 12. 0.75

Find an approximation, to the nearest tenth, for each square root. Then graph the square root on a number line.

13. $\sqrt{6}$ (0 1 2 3 4) 14. $\sqrt{11}$ (1 2 3 4 5) 15. $-\sqrt{24}$ (−7 −6 −5 −4 −3)

16. $\sqrt{30}$ (3 4 5 6 7) 17. $-\sqrt{38}$ (−8 −7 −6 −5 −4) 18. $\sqrt{51}$ (6 7 8 9 10)

19. $-\sqrt{65}$ (−10 −9 −8 −7 −6) 20. $\sqrt{72}$ (6 7 8 9 10) 21. $-\sqrt{89}$ (−11 −10 −9 −8 −7)

22. $\sqrt{118}$ (8 9 10 11 12) 23. $-\sqrt{131}$ (−13 −12 −11 −10 −9) 24. $\sqrt{104}$ (8 9 10 11 12)

Determine whether each number is rational or irrational. If it is irrational, find two consecutive integers between which its graph lies on the number line.

25. $\sqrt{28}$ 26. $-\sqrt{9}$ 27. $\sqrt{56}$

28. $-\sqrt{14}$ 29. $\sqrt{36}$ 30. $\sqrt{99}$

31. $-\sqrt{73}$ 32. $\sqrt{196}$ 33. $\sqrt{77}$

34. $-\sqrt{100}$ 35. $\sqrt{88}$ 36. $-\sqrt{46}$

14–2 NAME ______________________ DATE ________ PERIOD ______

Study Guide

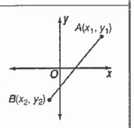
Student Edition Pages 606–611

The Distance Formula

You can use the Distance Formula to find the distance between two points on the coordinate plane.

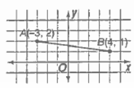
The Distance Formula

The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Example: Find the distance between $A(-3, 2)$ and $B(4, 1)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - (-3))^2 + (1 - 2)^2}$$
$$= \sqrt{7^2 + (-1)^2}$$
$$= \sqrt{49 + 1}$$
$$= \sqrt{50} \text{ or about 7.1 units}$$



Find the distance between each pair of points. Round to the nearest tenth, if necessary.

1. $P(-4, 0)$, $Q(5, 0)$
2. $X(0, 0)$, $Y(6, 8)$
3. $J(-5, 1)$, $K(-2, 5)$ see notes
4. $M(-4, -14)$, $N(3, 10)$
5. $R(-7, 4)$, $S(2, -1)$
6. $C(0, -5)$, $D(3, 2)$
7. $X(5, 9)$, $Y(2, 3)$ see notes
8. $A(-9, 1)$, $B(-8, 3)$
9. $G(5, -4)$, $H(10, 6)$
10. $U(7, -3)$, $V(-3, -2)$
11. $M(1, 3)$, $N(-1, 4)$
12. $X(12, -3)$, $Y(7, -15)$
13. $A(1, 20)$, $B(12, -4)$ see notes
14. $E(-3, -3)$, $F(-8, -8)$
15. $A(5, 4)$, $B(0, 6)$ see notes
16. $S(-6, -6)$, $T(-5, -1)$

3) $(-5, 1)$
$(-2, 5)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-2 + +5)^2 + (5 \mp 1)^2}$$

$$d = \sqrt{(+3)^2 + (+4)^2}$$

$$d = \sqrt{9 + 16}$$

$$d = \sqrt{25}$$

$$\boxed{d = 5 \text{ units}}$$

7) $(5, 9)$
$(2, 3)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(5-2)^2 + (9-3)^2}$$

$$d = \sqrt{(3)^2 + (6)^2}$$

$$d = \sqrt{9 + 36}$$

$$d = \sqrt{45}$$

$$d \approx 6.7 \text{ units}$$

13) $(1, 20)$ $(12, -4)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(12-1)^2 + (-4-20)^2$

11^2 -24^2

$121 + 576$ 26 27

676 729

$\sqrt{697} \approx 26$ units

15) $(5,4)$ $(0,6)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(+5 + {}^-0)^2 + (+4 + {}^-6)^2}$

$d = \sqrt{(5)^2 + (-2)^2}$

$d = \sqrt{25 + 4}$

$d = \sqrt{29} \approx 5.4$ units

14-2 NAME ______________________ DATE ________ PERIOD ______

Practice

Student Edition Pages 606–611

The Distance Formula

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

1. $X(4, 2)$, $Y(8, 6)$
2. $Q(-3, 8)$, $R(2, -4)$
3. $A(0, -3)$, $B(-6, 5)$
4. $M(-9, -5)$, $N(-4, 1)$
5. $J(6, 2)$, $K(-7, 5)$
6. $S(-2, 4)$, $T(-3, 8)$
7. $V(-1, -2)$, $W(-9, -7)$
8. $O(5, 2)$, $P(7, -4)$
9. $G(3, 4)$, $H(-2, 1)$

Find the value of a if the points are the indicated distance apart.

10. $C(1, 1)$, $D(a, 7)$; $d = 10$
see notes
11. $Y(a, 3)$, $Z(5, -1)$; $d = 5$
12. $F(3, -2)$, $G(-9, a)$; $d = 13$
see notes
13. $W(-2, a)$, $X(7, -4)$; $d = \sqrt{85}$
see notes
14. $B(a, -6)$, $C(8, -3)$; $d = \sqrt{34}$
15. $T(2, 2)$, $U(a, -4)$; $d = \sqrt{72}$

10) $(1, 1)$ $(A, 7)$ $d = 10$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(10)^2 = \left(\sqrt{(1-A)^2 + (1-7)^2}\right)^2$$

$$100 = (1-A)^2 + (1+-7)^2$$

$$100 = (1-A)^2 + (-6)^2$$

$$100 = (1-A)^2 + \cancel{36}$$

$$+-36 = \qquad +-\cancel{36}$$

$$\sqrt{64} = \sqrt{(1-A)^2}$$

$$\pm 8 = \cancel{1} - A$$

$$+-1 = \cancel{+-1}$$

$$\pm 8 + -1 = -A$$

$$-A = 8 + -1 \quad \text{or} \quad -8 + -1$$

$$\frac{-A}{-1} = \frac{+7}{-1} \quad \text{or} \quad \frac{-9}{-1}$$

$$A = -7 \quad \text{or} \quad A = +9$$

12) $(3,-2)$ $(9, A)$ $d = 13$ units

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$13 = \sqrt{(3 - -9)^2 + (-2 - A)^2}$

$13 = \sqrt{(3+9)^2 + (-2+-A)^2}$

$13 = \sqrt{(12)^2 + (-2+-A)^2}$

$(13)^2 = \left(\sqrt{144 + (-2+-A)^2}\right)^2$

$169 = \cancel{144} + (-2+-A)^2$

$+-144 = +-\cancel{144}$

$\sqrt{25} = \sqrt{(-2+-A)^2}$

$\pm 5 = \cancel{-2}+-A$

$+2 = \cancel{+2}$

$2 \pm 5 = -A$

7 or $-3 = \cancel{-1}A$

-1 -1 $\cancel{-1}$

-7 or $+3 = A$

$A = -7$ or $A = +3$

$(9,-7)$ or $(9,+3)$

13) $(-2, A)$ $(7,-4)$ $d = \sqrt{85}$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$\sqrt{85} = \sqrt{(-2-7)^2 + (A - -4)^2}$

$(\sqrt{85})^2 = \left(\sqrt{(-9)^2 + (A+4)^2}\right)^2$

$85 = (-9)^2 + (A+4)^2$

$85 = \cancel{81} + (A+4)^2$

$+-81 = +-\cancel{81}$

$\sqrt{4} = \sqrt{(A+4)^2}$

$\pm 2 = A\cancel{+4}$

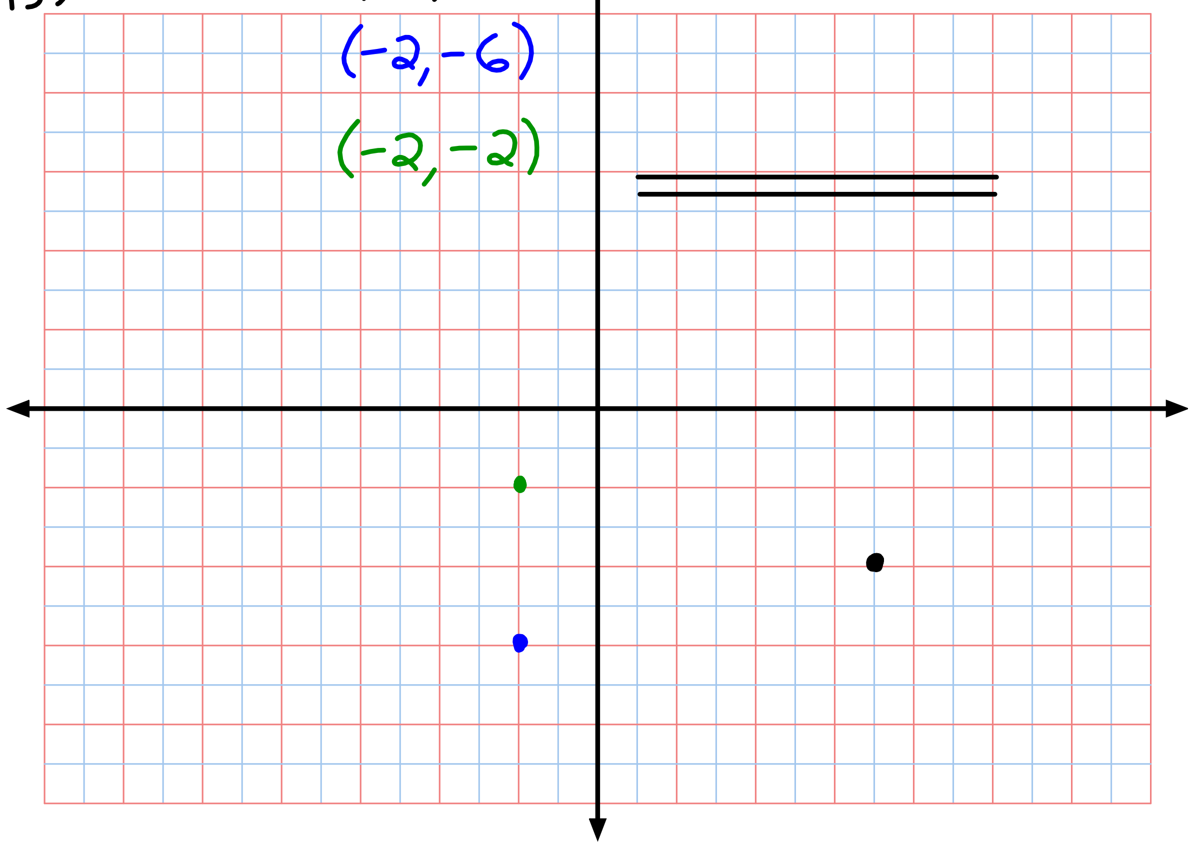
$+-4 = \cancel{+-4}$

$A = +2+-4$ or $A = -2+-4$

$A = -2$ or $A = -6$

$(-2,-2)$ or $(-2,-6)$

13) cont'd $(-2, A)$ $(7, -4$



14-3 NAME ______________ DATE ______ PERIOD ______

Study Guide

Student Edition Pages 614–619

Simplifying Radical Expressions

To simplify a radical expression such as $\sqrt{54}$, first identify any perfect square factors of the radicand. Then apply the Product Property of Square Roots.

$\sqrt{54} = \sqrt{6 \cdot 9}$ — *$3^2 = 9$, so 9 is a perfect square factor of 54.*
$= \sqrt{6} \cdot \sqrt{9}$ — *Product Property of Square Roots*
$= \sqrt{6} \cdot 3$ or $3\sqrt{6}$ — *Simplify $\sqrt{9}$.*

To simplify radical expressions such as $\frac{\sqrt{24}}{\sqrt{2}}$ and $\frac{\sqrt{6}}{\sqrt{5}}$ that involve division, you must eliminate the radicals in the denominator. To do so, you can use the Quotient Property of Square Roots and a method called **rationalizing the denominator**. Rationalizing the denominator involves multiplying the fraction by a special form of 1.

Examples: Simplify each expression.

a. $\frac{\sqrt{24}}{\sqrt{2}}$

$\frac{\sqrt{24}}{\sqrt{2}} = \sqrt{\frac{24}{2}}$ — *Quotient Property*
$= \sqrt{12}$ — *$24 \div 2 = 12$*
$= \sqrt{3 \cdot 2^2}$ — *$3 \cdot 4 = 3 \cdot 2^2$*
$= 2\sqrt{3}$ — *$\sqrt{2^2} = 2$*

b. $\frac{\sqrt{6}}{\sqrt{5}} =$

$\frac{\sqrt{6}}{\sqrt{5}} = \frac{\sqrt{6}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ — *$\frac{\sqrt{5}}{\sqrt{5}} = 1$*
$= \frac{\sqrt{6 \cdot 5}}{\sqrt{5 \cdot 5}}$ — *Product Property*
$= \frac{\sqrt{30}}{\sqrt{25}}$ — *Quotient Property*
$= \frac{\sqrt{30}}{5}$ — *Simplify.*

Simplify each expression. Leave in radical form, if necessary.

1. $\sqrt{28}$ — $\sqrt{28}$: $7 \cdot 4$, $2 \cdot 2$ — $2\sqrt{7}$
2. $\sqrt{48}$ — 48: 3 $2 \cdot 2$ $2 \cdot 2$ — $4\sqrt{3}$
3. $\sqrt{50}$ — 50: 2 $5 \cdot 5$ — $5\sqrt{2}$
4. $\sqrt{8}$ — 8: $4 \cdot 2$, $2 \cdot 2$ — $2\sqrt{2}$
5. $\sqrt{99}$ — 99: $9 \cdot 11$, $3 \cdot 3$ — $3\sqrt{11}$
6. $\sqrt{12} \cdot \sqrt{3}$ — see notes
7. $\sqrt{8} \cdot \sqrt{6}$ — see notes
8. $2\sqrt{3} \cdot \sqrt{3}$ — see notes
9. $\frac{\sqrt{18}}{\sqrt{6}} = \sqrt{\frac{18}{6}} = \sqrt{3}$
10. $\frac{\sqrt{75}}{\sqrt{3}} = \sqrt{\frac{75}{3}} = \sqrt{25} = 5$
11. $\frac{\sqrt{49}}{\sqrt{7}}$
12. $\frac{\sqrt{72}}{\sqrt{36}}$
13. $\frac{\sqrt{3}}{\sqrt{4}} = \sqrt{\frac{3}{4}}$ see notes
14. $\frac{\sqrt{5}}{\sqrt{3}}$
15. $\frac{\sqrt{12}}{\sqrt{5}}$
16. $\frac{\sqrt{6}}{\sqrt{8}}$

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6) $\sqrt{12} \cdot \sqrt{3} \rightarrow \sqrt{36} \rightarrow 6$

$2 \cdot 2 \cdot 3 \quad 3$

$2 \cdot 3 \rightarrow 6$

7) $\sqrt{8} \cdot \sqrt{6} \rightarrow \sqrt{48}$

$2 \cdot 2 \cdot 2 \quad 2 \cdot 3$

$2 \cdot 2$

$4\sqrt{3}$

8) $2\sqrt{3} \cdot \sqrt{3} \rightarrow 2\sqrt{3 \cdot 3} = 2(3) = 6$

$1 \cdot 3 \quad 1 \cdot 3$

$3 \cdot 3$

$\sqrt{9} = 3$

$2 \cdot 3$

6

13) $\frac{\sqrt{3}}{\sqrt{4}} = \cancel{\sqrt{\frac{3}{4}}}$

$\frac{\sqrt{3}}{\sqrt{4}} \cdot \frac{\sqrt{4}}{\sqrt{4}} \rightarrow \frac{\sqrt{3 \cdot 4}}{4} = \frac{\sqrt{3 \cdot 2 \cdot 2}}{4} = \frac{\overset{1}{\cancel{2}}\sqrt{3}}{\underset{2}{\cancel{4}}}$

$\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

14-3 NAME ____________ DATE ______ PERIOD ______

Practice

Student Edition Pages 614–619

Simplifying Radical Expressions

Simplify each expression. Leave in radical form.

1. $\sqrt{28}$
2. $\sqrt{48}$
3. $\sqrt{72}$ 72 → 9 · 8 → 3 3 2 2 2; 3 · 2 = 6; $6\sqrt{2}$
4. $\sqrt{90}$
5. $\sqrt{175}$
6. $\sqrt{245}$
7. $\sqrt{7} \cdot \sqrt{14} = \sqrt{7 \cdot 2 \cdot 7}$; 7; 7 · 2 · 7; $7\sqrt{2}$
8. $\sqrt{2} \cdot \sqrt{10}$
9. $\sqrt{10} \cdot \sqrt{60}$
10. $\frac{\sqrt{48}}{\sqrt{2}}$
11. $\frac{\sqrt{54}}{\sqrt{3}}$
12. $\frac{\sqrt{96}}{\sqrt{8}}$
13. $\frac{\sqrt{20}}{\sqrt{3}}$ see notes
14. $\frac{\sqrt{2}}{\sqrt{10}}$
15. $\frac{\sqrt{8}}{\sqrt{6}}$
16. $\frac{5}{4 - \sqrt{7}}$
17. $\frac{4}{3 + \sqrt{2}}$ see notes
18. $\frac{3}{3 - \sqrt{3}}$ see notes

Simplify each expression. Use absolute value symbols if necessary.

19. $\sqrt{50x^2}$
20. $\sqrt{27ab^3}$
21. $\sqrt{49c^6d^4}$ see notes
22. $\sqrt{63x^2y^5z^2}$
23. $\sqrt{56m^2n^4p^3}$
24. $\sqrt{108r^2s^3t^6}$ see notes

13) $\frac{\sqrt{20}}{\sqrt{3}} = \cancel{\sqrt{\frac{20}{3}}}$

20 → 2 · 2 · 5 ; 3 → 3

$$\frac{\sqrt{20}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{\sqrt{20 \cdot 3}}{\sqrt{3 \cdot 3}} = \frac{\sqrt{2 \cdot 2 \cdot 5 \cdot 3}}{3} = \boxed{\frac{2\sqrt{15}}{3}}$$

17) $\dfrac{4}{3+\sqrt{2}} \cdot \dfrac{(3-\sqrt{2})}{(3-\sqrt{2})}$

$$\frac{4(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$$

$$\frac{12-4\sqrt{2}}{9+\cancel{^-3\sqrt{2}+3\sqrt{2}}+{}^-\underbrace{\sqrt{2}\cdot\sqrt{2}}_{\sqrt{2\cdot 2}\,=\,\sqrt{4}}}$$

$$\frac{12-4\sqrt{2}}{9+{}^-2}$$

$$\boxed{\frac{12-4\sqrt{2}}{7}}$$

18) $\dfrac{3}{(3-\sqrt{3})} \cdot \dfrac{(3+\sqrt{3})}{(3+\sqrt{3})}$

$$\frac{3(3+\sqrt{3})}{(3+{}^-\sqrt{3})(3+\sqrt{3})} = \frac{9+3\sqrt{3}}{9+3\sqrt{3}-3\sqrt{3}-\underbrace{\sqrt{3}\cdot\sqrt{3}}_{\sqrt{9}\,=\,3}}$$

$$= \frac{9+3\sqrt{3}}{9+\cancel{3\sqrt{3}}-\cancel{3\sqrt{3}}-3}$$

$$= \frac{9+3\sqrt{3}}{6}$$

21) $\sqrt{49c^6d^4}$

$7 \cdot 7 \quad c \cdot c \quad c \cdot c \quad c \cdot c \quad d \cdot d \quad d \cdot d$

$7c^3d^2$

24) $\sqrt{108r^2s^3t^6}$

9 12

3 3 2 2 3

2 2 3 3 3

$2 \quad 54$

$2 \quad 27$

$3 \quad 9$

$3 \quad 3$

$2 \; 2 \quad 3 \; 3 \quad 3$

$r \cdot r \quad s \cdot s \cdot s \quad t \cdot t \quad t \cdot t \quad t \cdot t$

$t \cdot t \cdot t$

$t^3 \cdot s \cdot r \cdot 6\sqrt{3 \cdot s}$

$6rst^3\sqrt{3s}$

NAME ______ DATE ______ PERIOD ______

14-4 **Study Guide**

Student Edition Pages 620–623

Adding and Subtracting Radical Expressions

You can use the Distributive Property to add and subtract radical expressions with the same radicand.

Example 1: Simplify each expression.

a. $2\sqrt{3} + 8\sqrt{3}$

$2\sqrt{3} + 8\sqrt{3} = (2 + 8)\sqrt{3}$ *Distributive Property*

$= 10\sqrt{3}$ *Simplify.*

b. $15\sqrt{10} - 2\sqrt{10}$

$15\sqrt{10} - 2\sqrt{10} = (15 - 2)\sqrt{10}$ *Distributive Property*

$= 13\sqrt{10}$ *Simplify.*

Recall that when you add monomials, only like terms can be combined. The same is true when you add or subtract radical expressions. Radical expressions are like terms if they have the same radicand when they are in simplest form.

Example 2: Simplify $4\sqrt{2} + 6\sqrt{7} - 11\sqrt{7}$.

$4\sqrt{2} + 6\sqrt{7} - 11\sqrt{7} = 4\sqrt{2} + (6\sqrt{7} - 11\sqrt{7})$ *Group like terms.*

$= 4\sqrt{2} + (6 - 11)\sqrt{7}$ *Distributive Property*

$= 4\sqrt{2} - 5\sqrt{7}$ *Simplify.*

Simplify each expression.

1. $8\sqrt{5} + 8\sqrt{5} = 16\sqrt{5}$
2. $5\sqrt{11} - 3\sqrt{11} = 2\sqrt{11}$
3. $-9\sqrt{2} + 1\sqrt{2} = -8\sqrt{2}$
4. $-3\sqrt{3} - 10\sqrt{3} = -13\sqrt{3}$
5. $4\sqrt{6} + 1\sqrt{6} = 5\sqrt{6}$
6. $1\sqrt{10} - 6\sqrt{10} + 7\sqrt{10} = 2\sqrt{10}$
7. $2\sqrt{2} - 5\sqrt{2} + 4\sqrt{5} = -3\sqrt{2} + 4\sqrt{5}$
8. $1\sqrt{11} - 15\sqrt{3} - 10\sqrt{3} = \sqrt{11} - 25\sqrt{3}$
9. $8\sqrt{13} + 3\sqrt{13} - 4\sqrt{7} - 3\sqrt{7}$
10. $-4\sqrt{6} + 2\sqrt{6} + \sqrt{3} + \sqrt{3}$

$-2\sqrt{6} + 2\sqrt{3}$

11. $-3\sqrt{5} + 9\sqrt{2} + 5\sqrt{2} + 5\sqrt{5}$
12. $7\sqrt{2} + \sqrt{8}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

13. $3\sqrt{3} + \sqrt{27}$

$3 \cdot 3 \cdot 3$

$3\sqrt{3} + 3\sqrt{3}$

$6\sqrt{3}$

14. $5\sqrt{32} - 6\sqrt{2}$

$2 \cdot 2 \cdot 5\sqrt{2} - 6\sqrt{2}$

$20\sqrt{2} - 6\sqrt{2}$

$14\sqrt{2}$

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14-4 **Practice**

Student Edition Pages 620–623

Adding and Subtracting Radical Expressions

Simplify each expression.

1. $3\sqrt{7} + 4\sqrt{7}$
2. $9\sqrt{2} - 4\sqrt{2}$
3. $-5\sqrt{17} + 12\sqrt{17}$
4. $7\sqrt{3} - 3\sqrt{3}$
5. $-8\sqrt{5} + 2\sqrt{5}$
6. $-7\sqrt{11} - 2\sqrt{11}$
7. $13\sqrt{10} - 5\sqrt{10}$
8. $-6\sqrt{7} + 4\sqrt{7}$
9. $3\sqrt{7} + \sqrt{3}$

simplest form

10. $2\sqrt{6} + 4\sqrt{6} + 5\sqrt{6}$
11. $5\sqrt{3} + 4\sqrt{3} - 7\sqrt{3}$
12. $3\sqrt{2} - 2\sqrt{2} + 5\sqrt{2}$
13. $11\sqrt{5} - 3\sqrt{5} - 2\sqrt{5}$
14. $6\sqrt{13} + 3\sqrt{13} - 12\sqrt{13}$

$-3\sqrt{13}$

15. $4\sqrt{10} - 3\sqrt{10} - 5\sqrt{10}$
16. $4\sqrt{6} - 2\sqrt{6} + 3\sqrt{6}$
17. $7\sqrt{7} + 4\sqrt{3} - 5\sqrt{7}$
18. $-9\sqrt{2} + 4\sqrt{6} + 2\sqrt{2}$
19. $\sqrt{12} + 2\sqrt{27}$
20. $5\sqrt{63} - \sqrt{28}$
21. $-4\sqrt{96} + 6\sqrt{24}$
22. $-3\sqrt{45} + 3\sqrt{180}$

see notes

23. $-4\sqrt{56} + 3\sqrt{126}$
24. $2\sqrt{72} - 3\sqrt{50}$
25. $7\sqrt{32} + 3\sqrt{75}$
26. $\sqrt{32} + \sqrt{8} + \sqrt{18}$
27. $2\sqrt{20} - \sqrt{80} + \sqrt{45}$

see notes

22) $-3\sqrt{45} + 3\sqrt{180}$

$9 \cdot 5$ $\quad$ $18 \quad 10$

$3 \cdot 3$ $\quad$ $3 \quad 6 \quad 2 \quad 5$

$3 \quad 2$

$-9\sqrt{5} + 18\sqrt{5}$

$9\sqrt{5}$

27) $2\sqrt{20} - \sqrt{80} + \sqrt{45}$

$4 \quad 5$ $\quad$ $8 \quad 10$ $\quad$ $5 \quad 9$

$2 \quad 2$ $\quad$ $2 \quad 2 \quad 2 \quad 2 \quad 5$ $\quad$ $3 \quad 3$

$4\sqrt{5} + {}^{-}4\sqrt{5} + 3\sqrt{5}$

$3\sqrt{5}$

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Study Guide

Student Edition
Pages 624–629

Solving Radical Equations

Equations that contain radicals are called **radical equations**.

Steps for Solving Radical Equations

1. Isolate the radical on one side of the equation.
2. Square each side of the equation to eliminate the radical.
3. Check all solutions. Reject any solutions that do not satisfy the original equation.

Examples: Solve each equation. Check your solution.

a. $\sqrt{x} - 2 = 7$

$\sqrt{x} - 2 = 7$

$\sqrt{x} - 2 + 2 = 7 + 2$ *Add 2 to each side.*

$\sqrt{x} = 9$

$(\sqrt{x})^2 = 9^2$ *Square each side.*

$x = 81$

Check: $\sqrt{x} - 2 = 7$

$\sqrt{81} - 2 \stackrel{?}{=} 7$ *Replace x with 81.*

$9 - 2 \stackrel{?}{=} 7$

$7 = 7$ ✓

The solution is 81.

b. $\sqrt{x+1} + 5 = 4$

$\sqrt{x+1} + 5 = 4$

$\sqrt{x+1} = -1$ *Subtract 5.*

$(\sqrt{x+1})^2 = (-1)^2$ *Square each side.*

$x + 1 = 1$

$x = 0$

Check: $\sqrt{x+1} + 5 = 4$

$\sqrt{0+1} + 5 \stackrel{?}{=} 4$

$1 + 5 \stackrel{?}{=} 4$

$6 \neq 4$

There is no solution.

Some radical equations have no solution when the domain is the set of real numbers. Example b has no solution because the square root of $x + 1$ cannot be negative.

Solve each equation. Check your solution.

1. $\sqrt{x} = 7$
2. $\sqrt{x} = 2$
3. $\sqrt{x} = -9$
4. $\sqrt{x} - 9 = -1$
5. $\sqrt{x} + 12 = 6$ see notes
6. $\sqrt{x} + 2 = 4$
7. $\sqrt{x} - 8 = -3$
8. $\sqrt{x} - 1 = -5$ see notes
9. $\sqrt{x+4} = 3$
10. $2 = \sqrt{x-5}$
11. $\sqrt{x-2} + 1 = 6$
12. $\sqrt{x+3} - 7 = 0$ see notes

5) $\sqrt{x} \cancel{+12} = 6$

$\cancel{-12} = -12$

$(\cancel{\sqrt{x}})^{\cancel{2}} = (-6)^2$

$x = 36$

$18 \neq 6$

No solution

$\varnothing$

8) $\left(\cancel{\sqrt{x-1}}\right)^{\cancel{2}} = (-5)^2$

$x \cancel{+1} = 25$
$\cancel{+^-1} = +^+1$

$x = 26$

check $\sqrt{x-1} = -5$

$\sqrt{(26)-1} \stackrel{?}{=} -5$

$\sqrt{25} \stackrel{?}{=} -5$

$5 \stackrel{?}{\neq} -5$

No Solution

$\varnothing$

$\sqrt{x} \cancel{+^-1} = -5$
$\cancel{+^+1} = +^+1$

$\left(\cancel{\sqrt{x}}\right)^{\cancel{2}} = (-4)^2$

$x = 16$

check $\sqrt{x} + -1 = -5$

$\sqrt{(16)} + -1 \stackrel{?}{=} -5$

$+4 + -1 \stackrel{?}{=} -5$

$+3 \neq -5$

No solution

$\varnothing$

12) $\sqrt{x+3} \cancel{+^-7} = 0$
$\cancel{+^+7} = +^+7$

$\left(\cancel{\sqrt{x+3}}\right)^2 = (7)^2$

$x \cancel{+3} = 49$
$\cancel{+^-3} = +^-3$

$x = ^+46$

Check: $\sqrt{x+3} - 7 = 0$

$\sqrt{(46)+3} - 7 \stackrel{?}{=} 0$

$\sqrt{49} - 7 \stackrel{?}{=} 0$

$7 +^- 7 \stackrel{?}{=} 0$

$0 = 0$ ✓

$x = 46$

14-5

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Practice

Student Edition
Pages 624–629

Solving Radical Equations

Solve each equation. Check your solution.

1. $\sqrt{x} - 6 = 3$
2. $\sqrt{k} + 7 = 20$
3. $\sqrt{p} + 3 = 3$
4. $\sqrt{n + 11} = 5$
5. $\sqrt{w - 2} - 1 = 6$
6. $\sqrt{y - 5} + 9 = 14$
7. $\sqrt{2r + 1} - 10 = -1$
8. $\sqrt{3h - 11} + 2 = 9$ see notes
9. $\sqrt{a + 4} = a - 8$ see notes
10. $\sqrt{z - 3} + 5 = z$
11. $\sqrt{3b + 9} + 3 = b$ see notes
12. $\sqrt{5f - 5} + 1 = f$
13. $\sqrt{8 + 2c} = c - 8$
14. $\sqrt{3s - 6} = s - 2$
15. $\sqrt{4h + 4} + h = 7$
16. $\sqrt{5m + 4} = m + 2$ see notes
17. $\sqrt{2y - 7} - y = -5$
18. $\sqrt{3k + 4} + k = 8$ see notes

8) $\sqrt{3h-11} \cancel{+2} = 9$

$\cancel{+^-2} = +^-2$

$\left(\cancel{\sqrt{3h-11}}\right)^{\cancel{2}} = \left(^+7\right)^2$

$3h \cancel{-11} = 49$

$+\cancel{^+11} = +^+11$

$\dfrac{\cancel{3}h}{\cancel{3}} = \dfrac{60}{3}$

$h = 20$ ✓

9) $\left(\sqrt{A+4}\right)^2 = (1A - 8)^2$

$(A-8)(A-8)$

$A+4 = A^2 - 8A + -8A + {}^{+}64$

$A + \cancel{4} = A^2 + -16A + 64$

$+\cancel{-4} = \qquad + -4$

$\cancel{A} = A^2 + -16A + 60$

$+\cancel{-A} = \qquad + -1A$

$0 = A^2 + -17A + 60$

10 6
2 30
12 5
3 20
1 60

$0 = (A - 12)(A - 5)$

$A \cancel{+12} = 0$ or $A \cancel{+5} = 0$

$\cancel{+12} = {}^{+}12$ $\quad$ $\cancel{+5} = +5$

$A = 12$ ✓ or $\cancel{A = 5}$

$\cancel{3 \neq 3}$

11) $\sqrt{3B+9} \cancel{+3} = B$

$\cancel{+-3} = +-3$

$\left(\sqrt{3B+9}\right)^{\cancel{2}} = (B+-3)^2$

$3B+9 = \underbrace{(B+-3)^2}$

$3B+9 = B^2 + -6B \quad +9$

16) $(\sqrt{5m+4})^2 = (m+2)^2$

$5m + \cancel{4} = m^2 + \cancel{4} + 4m$
$\quad + \cancel{-4} \qquad \cancel{+-4}$

$\cancel{5m} = m^2 + 4m$
$+\cancel{-5m} \qquad +-5m$

$0 = m^2 + -1m$

$0 = m(m + -1)$

$\sqrt{}\, m = 0$ or $m = 1 \sqrt{}$

$0 = m + -1$
$+1 \quad +1$

$1 = m$

$\boxed{m = 0 \text{ or } 1}$

18) $\sqrt{3k+4} + k = 8$
$\qquad +-k \quad +-k$

$(\sqrt{3k+4})^2 = (8 + -k)^2$

$3k + 4 = 64 + k^2 + -16k$
$\quad + -4 \; + -4$

$3k = 60 + k^2 + -16k$
$-3k = \qquad +-3k$

$0 = 60 + k^2 + -19k$

$0 = k^2 + -19k + 60$

$0 = (k + -4)(k + -15)$

1	60
2	30
3	20
4	15
5	12
6	10

$k + -4 = 0$ or $k + -15 = 0$
$+4 \quad +4$ or $+15 \quad +15$

$\sqrt{}\, k = 4$ or $k \neq 15$