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Factors

Because $3 \times 4 = 12$, we say that 3 and 4 are **factors** of 12. In other words, factors are the numbers you multiply to get a product. Since $2 \times 6 = 12$, 2 and 6 are also factors of 12. The only factors of 5 are 1 and 5.

Numbers like 5 that have have exactly two factors, the number itself and 1, are called **prime numbers**.

Prime Number	2	3	29
Factors	1, 2	1, 3	1, 29
Products of Factors	1×2	1×3	1×29

Numbers like 12 that have more than two factors are called **composite numbers.**

Composite Number	4	12	33
Factors	1, 2, 4	1, 2, 3, 4, 6, 12	1, 3, 11, 33
Products of Factors	1×4 2×2	1×12 2×6 3×4	1×33 3×11

When two numbers are written as the product of their prime factors, they are in factored form.

Example 1: Write 45 in factored form.

$$45 = 9 \cdot 5$$
$$= 3 \cdot 3 \cdot 5 \quad \textit{Keep factoring until all factors are prime numbers.}$$

The factored form of 45 is $3 \cdot 3 \cdot 5$.

Example 2: Write $12x^2y$ in factored form.

$$12x^2y = 3 \cdot 4 \cdot x \cdot x \cdot y$$
$$= 3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y$$

The factored form of $12x^2y$ is $3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y$.

Find the factors of each number. Then classify each number as prime or composite.

1. 10
2. 7
3. 15 → composite 5×3 1, 3, 5, 15
4. 21
5. 31
6. 49 → composite 7×7 1, 7, 49
7. 47
8. 39

Factor each monomial.

9. $18a$
10. $35xy$ $5 \cdot 7 \cdot x \cdot y$
11. c^3 $c \cdot c \cdot c$
12. $20r^2$ $4 \cdot 5 \cdot r \cdot r$ $2 \cdot 2$ $2 \cdot 2 \cdot 5 \cdot r \cdot r$
13. $6y^2z$

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Factors

Find the factors of each number. Then classify each number as prime or composite.

1. 36
2. 31
3. 28
4. 70
5. 43
6. 27
7. 14
8. 97

Factor each monomial.

9. $30m^2n$
10. $-12x^2y^3$ → $-1 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$ $-4\ 3$ $2\ 2$
11. $-21ab^2$
12. $36r^3s$
13. $63x^3yz^2$
14. $-40pq^2r^2$ 40 $2 \cdot 20$ $4 \cdot 5$ $2 \cdot 2$ $-1 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot p \cdot q \cdot q \cdot r \cdot r$

Find the GCF of each set of numbers or monomials.

15. 27, 18 GCF = 9

9	27	18
1	3	2
	3	2

16. 9, 12
17. 45, 56
18. 4, 8, 16
19. 32, 36, 38 see notes
20. 24, 36, 48 see notes
21. $6x$, $9x$
22. $5y^2$, $15y$
23. $14c^2$, $-13d$ 1
24. $25mn^2$, $20m$
25. $12ab^2$, $18ab$
26. $-28x^2y^3$, $21xy^2$
27. $6xy$, $18y^2$

2	$6xy$	$18y^2$
3	$3xy$	$9y^2$
y	$1xy$	y^2
1	x	y
	x	y

GCF = $6y$

28. $18c^2d$, $27cd^2$
29. $7m$, mn

19) 32 36 38

2 | 32 36
2 | 16 18
 | 8 9
8 9

GCF = 4

2 | 4 38
 | 2 19
2 19

GCF = 2

20) 24 36 48

6 | 24 36
2 | 4 6
 | 2 3
2 3

GCF = 12

2 | 12 48
2 | 6 24
3 | 3 12
 | 1 4
1 4

GCF overall = 12

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Factoring Using the Distributive Property

When you use the Distributive Property to multiply a monomial by a polynomial, you show two factors and a product.

Distributive Property	Factors	Product
$3a(5a + 4) = 15a^2 + 12a$	$3a$ and $5a + 4$	$15a^2 + 12a$
$-2x(x^2 + 6x - 1) = -2x^3 - 12x^2 + 2x$	$-2x$ and $x^2 + 6x - 1$	$-2x^3 - 12x^2 + 2x$
$5rs(4r + 2s) = 20r^2s + 10rs^2$	$5rs$ and $4r + 2s$	$20r^2s + 10rs^2$

When you reverse the Distributive Property to identify the factors of the product, the polynomial is said to be in factored form. This is called **factoring** the polynomial.

Example: Factor $15ab^2 + 12a^2b^2$.

$15ab^2 = 3 \cdot 5 \cdot a \cdot b \cdot b$
$12a^2b^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b$

Begin by factoring each monomial. Then identify the factors both monomials have in common.

The common factors of $15ab^2$ and $12a^2b^2$ are $3 \cdot a \cdot b \cdot b$, so the greatest common factor is $3ab^2$.

$15ab^2 = 3ab^2(5)$
$12a^2b^2 = 3ab^2(4a)$

Now write each monomial as a product of $3ab^2$ and its other factors.

The factored form for $15ab^2 + 12a^2b^2$ is $3ab^2(5 + 4a)$.

Use the Distributive Property to check that the factored form is equivalent to the given polynomial.

Check: $3ab^2(5 + 4a) = 3ab^2(5) + 3ab^2(4a)$ or $15ab^2 + 12a^2b^2$ ✓

Factor each polynomial. If the polynomial cannot be factored, write prime.

1. $12a + 3b$ see notes
2. $8w + 6$
3. $15d^2 - 18d$
4. $5c^4 + 2c^2$
5. $12mn^3 - 5n$ see notes
6. $4g + 13h^3$ prime
7. $2 - 16x^2$
8. $35xy^3 + 7x^2y$
9. $27pw - 25q$ prime
10. $48c^2d^2 + 36c^2d$
11. $ad + 3x^2 + 9$ prime
12. $7gh^2 + 7g + 14gh$
13. $35a^2 + 15a - 20ab^2$ see notes

1) $3 \,|\, 12A + 3b$
$1 \,|\, 4A + 1b$

$3(4A + 1b)$

$3(4A + b)$

5) $n \,\big|\, 12mn^3 \mp {}^{-}5n$

$(12mn^2 + {-5})$

$n(12mn^2 + -5)$

13) $5A \,\big|\, 35A^2 + 15A \mp {}^{-}20AB^2$

${}^{+}7A^2 + {}^{+}3A + {}^{-}4AB^2$

$5A(7A + {}^{+}3 + {}^{-}4\ B^2)$

$5A(7A + 3 + {}^{-}4B^2)$

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Factoring Using the Distributive Property

Factor each polynomial. If the polynomial cannot be factored, write prime.

1. $4x + 16$	**2.** $3y^2 + 12y$	**3.** $10x + 5x^2y$
4. $7yz + 3x$	**5.** $15r + 20rs$	**6.** $14ab + 21a$
7. $9xy - 3xy^2$	**8.** $12m^2n - 18mn^2$	**9.** $8ab + 2a^2b^2$
10. $16a^2bc - 36ab^2$	**11.** $3x^2y + 25m^2$	**12.** $8x^2y^3 - 10xy$
13. $4xy^2 + 18xy + 14y$	**14.** $7m^2 + 28mn + 14n^2$	**15.** $2x^2y + 4xy - 2xy^2$
16. $3a^3b - 9a^2b + 15b^2$	**17.** $18a^2bc + 24ac^2 + 36a^3c$	**18.** $8x^3y^2 + 16xy + 28x^2y^3$

Find each quotient.

19. $(6m^2 + 4) \div 2$	**20.** $(14x^2 - 21x) \div 7x$ see notes
21. $(10x^2 + 15y^2) \div 5$	**22.** $(2c^2 + 4c) \div 2c$
23. $(12xy + 9y) \div 3y$	**24.** $(9a^2b - 27ab) \div 9ab$
25. $(25m^2n^2 + 15mn) \div 5mn$	**26.** $(3a^2b - 9abc^2) \div 3ab$

20)

$$7x \,\big|\, \underline{14x^2 \overset{-}{+} 21x}$$
$$2x \overset{-}{+} 3$$

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Study Guide

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Factoring Trinomials: $x^2 + bx + c$

To find the two binomial factors of a polynomial, use the FOIL method.

Example 1: Factor $x^2 + 5x + 6$.

The first term in the trinomial is x^2. Since $x \cdot x = x^2$, the first term of each binomial is x.

$$x^2 + 5x + 6 = (x + \blacksquare)(x + \blacksquare)$$

To find the last terms, find a number pair whose product is 6 and whose sum is 5.

Product	Factors	Sum
6	1, 6	$1 + 6 = 7$
6	2, 3	$2 + 3 = 5$ ✓

Therefore, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

Example 2: Factor $x^2 - 8x + 12$.

The first terms are both x. To find the last terms, find a number pair whose product is 12 and whose sum is -8.

Product	Factors	Sum
12	$-1, -12$	$-1 + (-12) = -13$
12	$-2, -6$	$-2 + (-6) = -8$ ✓
12	$-3, -4$	

Once the correct sum is found, it is not necessary to check any more factors. Therefore, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Example 3: Factor $x^2 - 2x - 15$.

The first terms are both x. To find the last terms, find a number pair whose product is -15 and whose sum is -2.

Product	Factors	Sum
-15	$1, -15$	$1 + (-15) = -14$
-15	$-1, 15$	$-1 + 15 = 14$
-15	$3, -5$	$3 + (-5) = -2$ ✓

Therefore, $x^2 - 2x - 15 = (x + 3)(x - 5)$.

Factor each trinomial.

1. $x^2 + 3x + 2$ $(x+1)(x+2)$
2. $w^2 + 6w + 9$ $\underbrace{(w+3)(w+3)}_{(w+3)^2}$
3. $r^2 + 14r + 24$
4. $z^2 - 6z + 5$
5. $f^2 - 6f + 8$
6. $x^2 - 15x + 56$
7. $v^2 + 15v + 36$
8. $k^2 - 23k + 42$
9. $y^2 - 20y + 100$ see notes
10. $a^2 + 4a - 45$ see notes
11. $x^2 + 7x - 18$
12. $m^2 - 21m - 22$ see notes

9) $y^2 + {}^-20y + {}^+100 \rightarrow$ 1, 100
2, 50
4, 25
5, 20
10, 10

$(y + {}^-10)(y + {}^-10)$

10) $A^2 + 4A + {}^{-}45$

1, 45
3, 15
5, 9

$(A + {}^{-}5)(A + 9)$

12) $m^2 + {}^{-}21m + {}^{-}22$

1, 22
2, 11

$(m + {}^{-}22)(m + 1)$

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10-3 **Practice**

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Factoring Trinomials: $x^2 + bx + c$

Factor each trinomial. If the trinomial cannot be factored, write prime.

1. $x^2 + 5x + 6$	2. $y^2 + 5y + 4$	3. $m^2 + 12m + 35$
4. $p^2 + 8p + 15$	5. $a^2 + 8a + 12$	6. $n^2 + 4n + 4$
7. $x^2 + 9x + 18$	8. $x^2 + x + 3$	9. $y^2 - 6y + 8$
10. $c^2 - 8c + 15$	11. $m^2 - 2m + 1$	12. $b^2 - 9b + 20$
13. $x^2 - 8x + 7$	14. $n^2 - 5n + 6$	15. $y^2 - 8y + 12$
16. $c^2 - 4c + 5$	17. $x^2 - x - 12$	18. $m^2 + 5m - 6$
19. $a^2 + 4a - 12$	20. $y^2 - y - 6$	21. $b^2 - 3b - 10$
22. $x^2 + 3x - 4$	23. $c^2 + 2c - 15$	24. $2x^2 + 10x + 8$ see notes
25. $3y^2 - 15y + 18$	26. $5m^2 - 10m - 40$ see notes	27. $3b^2 + 6b - 9$
28. $4n^2 + 12n + 8$	29. $2x^2 + 8x - 24$	30. $3y^2 - 15y + 12$

24) $2x^2 + 10x + 8 \rightarrow$ 1, 8
2, 4

$(2x + {}^{+}8)(x + {}^{+}1)$

or

$(2x + {}^{+}2)(x + {}^{+}4)$

26) $5m^2 \overset{-}{+} 10m \overset{-}{+} 40 \longrightarrow$

$\cancel{1, 40}$
2, 20
4, 10
5, 8

$(5m \overset{-}{+} 20)(1m + 2)$

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Factoring Trinomials: $ax^2 + bx + c$

To find the two binomial factors of a polynomial, use the FOIL method.

Example 1: Factor $5x^2 + 37x + 14$.

The first term in the trinomial is $5x^2$. The only factors of 5 are 5 and 1, so the first terms of the binomials are $5x$ and x.

$$5x^2 + 37x + 14 = (5x + \blacksquare)(x + \blacksquare)$$

The last term in the trinomial is 14, which has two pairs of factors, 1 and 14, and 2 and 7. Try the factor pairs until you find the one that gives a middle term of $37x$.

First Terms	Last Terms	Binomial Pair	Middle Term	Trinomial
$5x, x$	1, 14	$(5x + 1)(x + 14)$	$x + 70x = 71x$	$5x^2 + 71x + 14$
$5x, x$	14, 1	$(5x + 14)(x + 1)$	$14x + 5x = 19x$	$5x^2 + 19x + 14$
$5x, x$	2, 7	$(5x + 2)(x + 7)$	$2x + 35x = 37x$	$5x^2 + 37x + 14$ ✓

Therefore, $5x^2 + 37x + 14 = (5x + 2)(x + 7)$.

Example 2: Factor $6x^2 - 23x + 7$.

There are two possible factor pairs of the first term, $2x$ and $3x$, and $6x$ and x. The last term is positive. The sum of the inside and outside terms is negative. So, the factors of 7 are -1 and -7. Try the factor pairs until you find the one that gives a middle term of $-23x$.

First Terms	Last Terms	Binomial Pair	Middle Term	Trinomial
$2x, 3x$	$-1, -7$	$(2x - 1)(3x - 7)$	$-3x - 14x = -17x$	$6x^2 - 17x + 7$
$3x, 2x$	$-1, -7$	$(3x - 1)(2x - 7)$	$-2x - 21x = -23x$	$6x^2 - 23x + 7$ ✓

Therefore, $6x^2 - 23x + 7 = (3x - 1)(2x - 7)$.

Factor each trinomial.

1. $3x^2 + 4x + 1$ $(3x \overset{+}{+} 1)(1x \overset{+}{+} 1)$
2. $2w^2 + 3w + 1$
3. $2r^2 + 5r + 3$
4. $8z^2 + 14z + 5$
5. $5f^2 + 27f + 10$
6. $2x^2 - 3x + 1$
7. $7v^2 - 10v + 3$ see notes
8. $9k^2 - 9k + 2$ see notes
9. $4y^2 + 3y - 1$ see notes
10. $5a^2 + 6a - 8$ see notes

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9) $4y^2 + {}^{+}3y + {}^{-}1$

$(4y + {}^{-}1)(y + 1)$

or

$\cancel{(2y \quad 1)(2y \quad 1)}$

7) $7v^2 + {}^{-}10v + 3 \rightarrow 1, 3$

$(7v + {}^{-}3)(v + {}^{-}1)$

8) $9k^2 + {}^-9k + 2$

$(9k \quad 2)(k \quad 1)$ or $(9k \quad 1)(k \quad 2)$

or $(3k + {}^-2)(3k + {}^-1)$ $\cancel{(3k \quad 1)(3k \quad 2)}$

10) $5A^2 + {}^+6A + {}^-8$ → $\cancel{1, 8}$ $2, 4$

$(5A + {}^-4)(A + {}^+2)$

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10-4 Practice

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Factoring Trinomials: $ax^2 + bx + c$

Factor each trinomial. If the trinomial cannot be factored, write prime.

1. $2y^2 + 8y + 6$	2. $2x^2 + 5x + 2$	3. $3a^2 - 4a - 4$
4. $5m^2 - 4m - 1$	5. $2c^2 + 6c - 8$	6. $4q^2 + 2q + 3$
7. $3x^2 - 13x + 4$	8. $4y^2 - 14y + 6$	9. $2b^2 - b - 10$
10. $6a^2 + 8a + 2$	11. $3n^2 + 7n - 6$	12. $3x^2 - 3x - 6$
13. $2c^2 + 3c - 7$	14. $5y^2 - 17y + 6$	15. $2b^2 + 2b - 12$
16. $2x^2 + 10x + 8$	17. $3m^2 - 19m + 6$	18. $4a^2 + 10a - 6$
19. $7b^2 - 16b + 4$	20. $3y^2 - y - 10$	21. $6c^2 + 11c + 4$
22. $10x^2 - x - 2$ see notes	23. $12m^2 - 11m + 2$	24. $9y^2 - 3y - 6$
25. $8b^2 + 12b + 4$	26. $6x^2 + 8x - 8$	27. $4n^2 - 14n + 12$
28. $6x^2 + 18x + 12$	29. $4a^2 + 18a - 10$	30. $9y^2 - 15y + 6$

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22) $10x^2 \overset{-}{+} x \overset{-}{+} 2$

$\cancel{(5x \quad 1)(2x \quad 2)}$ or $\boxed{(5x+2)(2x\overset{-}{+}1)}$

$(10x \quad 1)(x \quad 2)$ or $(10x \quad 2)(x \quad 1)$

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10-5 Study Guide

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Special Factors

$$x^2 + 10x + 25 = (x + 5)(x + 5)$$

Trinomials like the one above that have two equal binomial factors are called **perfect square trinomials**. Recall that when a number is multiplied by itself, the result is a perfect square. For example, $4 \cdot 4 = 16$, so 16 is a perfect square.

Factoring Perfect Square Trinomials	Symbols: $a^2 + 2ab + b^2 = (a + b)(a + b)$ $a^2 - 2ab + b^2 = (a - b)(a - b)$ **Example:** $x^2 - 2x + 1 = (x - 1)(x - 1)$

Studying these properties of $x^2 + 10x + 25$ will help you factor other perfect square trinomials.

1. The first term, x^2, is a perfect square. $\quad x \cdot x = x^2$
2. The last term, 25, is a perfect square. $\quad 5 \cdot 5 = 25$
3. The middle term, $10x$, is twice the product of 5 and x. $\quad 2(5x) = 10x$

Example 1: Factor $y^2 - 14y + 49$.
The first term is a perfect square. The last term is a perfect square. The middle term is twice the product of the first and last terms.
So $y^2 - 14y + 49$ is a perfect square trinomial.
$y^2 - 14y + 49 = (y - 7)(y - 7)$ or $(y - 7)^2$

When two perfect squares are subtracted, the polynomial is called the **difference of two squares**. The difference of two squares also has a special pair of factors.

Factoring Differences of Squares	Symbols: $a^2 - b^2 = (a - b)(a + b)$ **Example:** $x^2 - 25 = (x - 5)(x + 5)$

( ) "quantity"

Example 2: Factor $g^2 - 36$.
$g^2 - 36$ is the difference of two squares.
$g^2 - 36 = (g - 6)(g + 6)$

$\cancel{y+8^2 \Rightarrow y+64}$

Factor each perfect square trinomial.

1. $y^2 + 16y + 64$ $\quad (y+8)(y+8) \quad (y+8)^2$
2. $a^2 + 14a + 49$
3. $25s^2 + 10s + 1$
4. $r^2 - 8r + 16$ $\quad (r-4)(r-4) \quad (r-4)^2$
5. $p^2 - 20p + 100$
6. $36h^2 - 12h + 1$
7. $4a^2 + 12a + 9$
8. $9v^2 + 24v + 16$

Factor each difference of squares.

9. $b^2 - 64$ $\quad (b+8)(b-8)$
10. $k^2 - 4$
11. $81 - x^2$
12. $36m^2 - 1$
13. $4y^2 - 9$
14. $4 - 25d^2$ $\quad (2+5d)(2-5d)$

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10-5 Practice

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Special Factors

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $y^2 + 6y + 9$
2. $x^2 - 4x + 4$
3. $n^2 + 6n + 3$
4. $m^2 - 12m + 36$
5. $y^2 - 10y + 20$
6. $4a^2 + 16a + 16$
7. $9x^2 + 6x + 1$
8. $4n^2 - 20n + 25$
9. $4y^2 + 9y + 9$

Determine whether each binomial is the difference of squares. If so, factor it.

10. $x^2 - 49$
11. $b^2 + 16$
12. $y^2 - 81$
13. $4m^2 - 9$
14. $9a^2 - 16$
15. $25r^2 + 9$
16. $18n^2 - 18$
17. $3x^2 - 12y^2$
18. $8m^2 - 18n^2$

Factor each polynomial. If the polynomial cannot be factored, write prime.

19. $4a - 24$
20. $6x + 9$
21. $x^2 + 5x - 10$
22. $2y^2 + 6y - 20$
23. $m^2 - 9n^2$
24. $a^2 - 8a + 16$
25. $5b^2 + 10b$
26. $9y^2 + 12y + 4$
27. $3x^2 - 3x - 18$

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