Lesson 9-3
Understand Experimental Probability

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I can...
determine the experimental probability of an event.

Solve & Discuss It!

Kevin is awarded a penalty shot. He will either score a goal or not score a goal. Are both outcomes equally likely? Explain.

Look for Relationships
What might affect the outcome?

Focus on math practices

Construct Arguments Lowe Senior High School's hockey team won 12, lost 5, and tied in 3 of their first 20 games this season. Which outcome is most likely for the team's next game? Explain your reasoning.

Essential Question How is experimental probability similar to and different from theoretical probability?

EXAMPLE 1 Compare Theoretical and Experimental Probability

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Talia and Yoshi plan for 1 out of 8, or 12.5%, of the players winning a prize. During the school fair, they kept track of the numbers of total players and winners and recorded the data in the table below. How does the actual number of winners compare to the expected number of winners?

Time Period	Total Players	Winners
10 A.M.–noon	213	22
Noon–2 P.M.	262	36
TOTAL	475	58

STEP 1 Determine the *relative frequency* of winners during each time period. The **relative frequency** is the ratio of the number of times an event occurs to the total number of trials.

Ratio of Winners to Total Players

Time Period	Total Players	Winners	Relative Frequency
10 A.M.–noon	213	22	$\frac{22}{213} \approx 10.3\%$
Noon–2 P.M.	262	36	$\frac{36}{262} \approx 13.7\%$
TOTAL	475	58	$\frac{58}{475} \approx 12.2\%$

STEP 2 The relative frequency of an event can also be called **experimental probability**. Compare the experimental probability based on the data to the theoretical probability of winning the game.

Theoretical Probability: $P(\text{red}) = \frac{1}{8} = 12.5\%$

Experimental Probability: $\frac{58}{475} \approx 12.2\%$

In the previous lesson, Talia and Yoshi expected about 63 winners for 500 players, based on theoretical probability. Based on the data, there were actually 58 winners out of 475 players.

The experimental probability is slightly lower than the theoretical probability of winning this game. There were slightly fewer winners than expected.

Try It!

During the second day of the school fair, Talia and Yoshi recorded 43 winners out of a total of 324 players. How does the actual number of winners compare to the expected number of winners?

Theoretical Probability: $P(\text{red}) = \frac{1}{8} = 12.5\%$

Experimental Probability: $\frac{\square}{324} \approx \square\%$

This experimental probability is ☐ than the theoretical probability.

There were ☐ winners than expected.

Convince Me! Will experimental probability always be close to theoretical probability? Explain.

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EXAMPLE 2 Use Experimental Probability to Make Predictions

Joaquin also kept track of players and winners for his game during the fair. Based on the results shown in the table, how many winners should he expect if 300 people play his game?

Use proportional reasoning to predict the number of likely winners, w, based on the experimental probability.

$$\frac{71}{416} = \frac{w}{300}$$

$$\frac{71}{416} \cdot 300 = \frac{w}{300} \cdot 300$$

$$51.2 \approx w$$

Joaquin should expect about 51 winners out of 300 players.

Time Period	Total Players	Winners	Relative Frequency
10 A.M.–noon	174	28	$\frac{28}{174} \approx 16.1\%$
Noon–2 P.M.	242	43	$\frac{43}{242} \approx 17.8\%$
TOTAL	416	71	$\frac{71}{416} \approx 17.1\%$

EXAMPLE 3 Explain Differences Between Theoretical and Experimental Probability

Amir and Marvin each flip a coin 50 times and record the result of each flip. The tables show their results.

Amir's Results	
Heads	Tails
26	24

Marvin's Results	
Heads	Tails
30	20

A. Based on theoretical probability, what are the expected results of 50 coin flips?

There are two possible outcomes—heads or tails—and both outcomes are equally likely. For each coin flip, the probability of landing heads up (or tails up) is 1 out of 2, or $\frac{1}{2}$.

After 50 flips, the results should be *about* 25 heads and 25 tails.

B. Why might their results be different from the expected results based on theoretical probability?

Theoretical probability can be used to estimate results, but does not guarantee results. The more times they flip their coins, the more likely it is that their results will be closer to the theoretical probability.

Try It!

Amir and Marvin continue until they each flip a coin 200 times. How do you expect Amir's results and Marvin's results to compare? How will their results compare with expected results based on theoretical probability?

synonyms → actually doing something

KEY CONCEPT

Relative frequency, or experimental probability, is based on the actual results of an experiment, while theoretical probability is based on calculated results from the knowledge of the possible outcomes. Experimental probability and theoretical probability may be close but are rarely exactly the same.

$$\text{Experimental probability} = \frac{\text{number of times an event occurs}}{\text{total number of times the experiment is carried out}}$$

This value changes each time an experiment is carried out.

The experimental probability tends to get closer to the theoretical probability of an experiment as more trials are conducted.

Do You Understand?

1. **Essential Question** How is experimental probability similar to and different from theoretical probability?

2. **Construct Arguments** How can experimental probability be used to make predictions?

3. **Reasoning** Is experimental probability always close to theoretical probability? Explain.

Do You Know How?

In 4–6, complete each statement.

Kelly flips a coin 20 times. The results are shown in the table, where "H" represents the coin landing heads up and "T" represents the coin landing tails up.

Flip	1	2	3	4	5
Result	H	T	T	H	H
Flip	6	7	8	9	10
Result	H	H	T	H	T
Flip	11	12	13	14	15
Result	H	T	T	T	H
Flip	16	17	18	19	20
Result	T	H	H	T	H

4. The theoretical probability that the coin will land heads up is ____.

5. Based on the data, the experimental probability that the coin will land heads up is ____.

6. The experimental probability is ____ than the theoretical probability.

Name: _____

Practice & Problem Solving

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Leveled Practice In 7 and 8, complete each statement.

7. The table shows the results of spinning a wheel 80 times. What is the relative frequency of the event "spin a 3"?

Wheel Spins				
Outcomes	1	2	3	4
Frequency	8	22	18	32

The relative frequency of the wheel landing on 3 is

$\frac{\text{number of times an event occurs}}{\text{total number of trials}} = \frac{\square}{\square} = \square\%$

18 out of 80 $= \frac{18 \div 2}{80 \div 2} = \frac{9}{40}$ → 22.5%

Theoretical Prob.

$\frac{1}{4} \rightarrow 0.25 \rightarrow 25\%$

8. Liz flips a coin 50 times. The coin lands heads up 20 times and tails up 30 times. Complete each statement.

The theoretical probability of the coin landing heads up is $\square$.

Based on Liz's results, the experimental probability of the coin landing heads up is $\square$.

The theoretical probability is $\square$ than the experimental probability in this experiment.

9. Jess spins a pointer 25 times and finds an experimental probability of the pointer landing on 3 to be $\frac{4}{25}$, or 16%. The theoretical probability of the spinner landing on 3 is $\frac{1}{4}$, or 25%. Why might there be a significant difference between the theoretical and experimental probabilities?

10. The table shows the results of a survey of 100 people randomly selected at an airport. Find the experimental probability that a person is going to City E.

Airport Destinations

Destination	Number of Responses
City A	28
City B	34
City C	16
City D	14
City E	8

11. The theoretical probability of selecting a consonant at random from a list of letters in the alphabet is $\frac{21}{26}$. Wayne opens a book, randomly selects a letter on the page, and records the letter. He repeats the experiment 200 times. He finds $P(\text{consonant}) = 60\%$. How does the theoretical probability differ from the experimental probability? What are some possible sources for this discrepancy?

12. **Higher Order Thinking** Seven different names are written onto sticks and placed into a cup. A stick is chosen 100 times, out of which the name Grace is chosen 23 times. How do the theoretical probability and experimental probability compare? Explain why there is a discrepancy between them, if there is any.

Exp $P(\text{Grace}) = \frac{23}{100} = 23\%$

Theo $P(\text{Grace}) = \frac{1}{7} = 0.1428 \approx 14\%$

13. Each of three friends flips a coin 36 times. Angel records "tails" 20 times. Michael records "tails" 17 times. Fernanda records "tails" 23 times.

a. Find the relative frequency with which each friend records "tails".

b. Which friend has a relative frequency that is closest to the theoretical probability of flipping "tails" 36 times? Explain.

Theo $\frac{18}{36}$

Angel $\frac{20}{36}$

Michael ✓ $\frac{17}{36}$

Fernanda $\frac{23}{36}$

Assessment Practice

14. In a survey, 125 people were asked to choose one card out of five cards labeled 1 to 5. The results are shown in the table. Compare the theoretical probability and experimental probability of choosing a card with the number 1.

Cards Chosen					
Number	1	2	3	4	5
Frequency	15	30	35	20	25

15. The best player on a basketball team makes 75% of all free throws. The second-best player makes 70% of all free throws, and the third-best player makes 65% of all free throws. Based on their experimental probabilities, estimate the number of free throws each player will make in his or her next 40 attempts. Explain.