


**Explore It!**

The intersecting skis form four angles.



**Lesson 8-4**  
Solve Problems Using Angle Relationships

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**I can...**  
solve problems involving angle relationships.

**A.** List all the pairs of angles that share a ray.

**B.** Suppose the measure of  $\angle 1$  increases. What happens to the size of  $\angle 2$ ?  $\angle 3$ ?

**C.** How does the sum of the measures of  $\angle 1$  and  $\angle 2$  change when one ski moves? Explain.

**Focus on math practices**

**Construct Arguments** Why does the sum of all four angle measures stay the same when one of the skis moves?

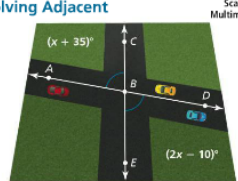
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**Essential Question** How are angles formed by intersecting lines related?

**EXAMPLE 1** Solve Problems Involving Adjacent and Vertical Angles

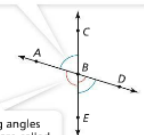
A skewed intersection has two roads that intersect at more than 20 degrees away from  $90^\circ$ . Determine whether the road intersection shown is skewed by finding the measures of  $\angle ABC$  and  $\angle DBE$ .

**Look for Relationships** What angle measures would a skewed intersection have?



Examine how the angles are related.

Angles opposite each other are called **vertical angles**. Vertical angles have equal measures.  $\angle ABC$  and  $\angle DBE$  are vertical angles.



Non-overlapping angles that share a ray are called **adjacent angles**.  $\angle ABE$  and  $\angle EBD$  are adjacent angles, sharing ray  $BE$ .

Write and solve an equation to find the value of  $x$ .

Read "m" as "the measure of" the named angle.

$$m\angle ABC = m\angle DBE$$

$$x + 35 = 2x - 10$$

$$x + 35 + 10 = 2x - 10 + 10$$

$$x + 45 = 2x$$

$$x - x + 45 = 2x - x$$

$$45 = x$$

Find the measure of an angle in the intersection.

$$m\angle ABC = (x + 35)^\circ$$

$$= (45 + 35)^\circ$$

$$= 80^\circ$$

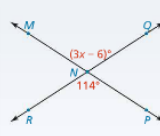
$\angle ABC$  and  $\angle DBE$  both measure  $80^\circ$ .

Since  $80^\circ$  is within  $20^\circ$  of  $90^\circ$ , the road intersection is not skewed.

**Try It!**

$\angle MNQ$  and  $\angle PNR$  are vertical angles. What is the value of  $x$ ?

Vertical angles are , so the equation  can be used to find  $x$ . The value of  $x$  is .



**Convince Me!** Why can you use an equation when solving for  $x$  in the diagram?

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**EXAMPLE 2** Solve Problems Involving Complementary and Supplementary Angles

a. Ray EG splits right angle DEF into two angles,  $\angle DEG$  and  $\angle GEF$ . Find the value of  $x$ .

b. The two angles shown are supplementary angles. Find the value of  $x$ .

**Complementary angles** have a sum of  $90^\circ$ .

$m\angle DEG + m\angle GEF = 90$   
 $3x + 36 = 90$   
 $3x + 36 - 36 = 90 - 36$   
 $3x = 54$   
 $\frac{3x}{3} = \frac{54}{3}$   
 $x = 18$

**Supplementary angles** have a sum of  $180^\circ$ .

$(\frac{x}{2} - 4) + 50 = 180$   
 $\frac{x}{2} + 46 = 180$   
 $\frac{x}{2} + 46 - 46 = 180 - 46$   
 $\frac{x}{2} = 134$   
 $2 \cdot \frac{x}{2} = 2 \cdot 134$   
 $x = 268$

**EXAMPLE 3** Find the Measure of an Unknown Angle

Find the measure of  $\angle PAR$ .

**STEP 1** Use vertical angles to find the value of  $x$ .

$m\angle UAG = m\angle KAR$   
 $6x = 2x + 16$   
 $6x - 2x = 2x - 2x + 16$   
 $4x = 16$   
 $\frac{4x}{4} = \frac{16}{4}$   
 $x = 4$

$m\angle UAG = (6x)^\circ = (6 \cdot 4)^\circ = 24^\circ$   
 So  $m\angle KAR = 24^\circ$ .

**STEP 2** Use complementary angles to find the measure of  $\angle PAR$ .

$m\angle KAR + m\angle PAR = 90^\circ$   
 $24^\circ + m\angle PAR = 90^\circ$   
 $m\angle PAR = 66^\circ$

**Try It!**

$m\angle 1$  is 4 times  $m\angle 2$ .  $\angle 1$  and  $\angle 2$  are complementary.  $\angle 1$  and  $\angle 3$  are vertical angles.  $\angle 3$  and  $\angle 4$  are supplementary. What are the measures of the four angles?

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**KEY CONCEPT**

This ray is shared by Angles 1 and 2, so they are **adjacent**. → next to

Angles 4 and 5 form a straight line and add up to  $180^\circ$ , so they are **supplementary**.

The sum of Angles 2 and 3 is  $90^\circ$ , so they are **complementary**.

Angles 1 and 4 are opposite each other, so they are **vertical**. → opposite angles → congruent

2 smaller angles that form a straight line

- Do You Understand?**
- Essential Question** How are angles formed by intersecting lines related?
  - Use Structure** Can vertical angles also be adjacent angles? Explain.
  - Reasoning** Do complementary and supplementary angles also have to be adjacent angles? Explain.

**Do You Know How?**

Use the diagram below for 4-6.

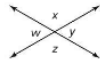
- List two pairs of adjacent angles.  
 $\angle 4$  and  $\angle 5$     $\angle 1$  and  $\angle 5$
- List all pairs of vertical angles.  
 $\angle 1$  and  $\angle 3$
- If  $\angle 1$  and  $\angle 3$  are the same measure, what is the value of  $x$ ?  
 $9x = 90$   
 $x = 10$

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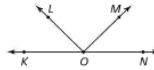
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**Practice & Problem Solving**

7. List each angle adjacent to  $\angle w$ .



8. List two pairs of adjacent angles.



9. Find the value of  $x$ .

Handwritten solution for problem 9:

$$9x + 90 + 18 = 180$$

$$9x + 108 = 180$$

$$-108 = -108$$

$$9x = 72$$

$$\frac{9x}{9} = \frac{72}{9}$$

$$x = 8$$

10. Find the value of  $x$ .

Handwritten solution for problem 10:

vertical (opposite) & congruent

$$125 = (5x + 30)$$

$$125 = 5x + 30$$

$$-30 = -30$$

$$95 = 5x$$

$$\frac{95}{5} = \frac{5x}{5}$$

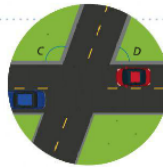
$$19 = x$$

$$x = 19$$

11.  $\angle 1$  and  $\angle 2$  are complementary angles. The measure of  $\angle 1$  is  $42^\circ$ . The measure of  $\angle 2$  is  $(3x)^\circ$ . Find the value of  $x$ .



12. Two streets form an intersection.  $\angle C$  and  $\angle D$  are supplementary angles. If the measure of  $\angle C$  is  $128^\circ$  and the measure of  $\angle D$  is two times the value of  $x$ , what is the value of  $x$ ?



13. If  $\angle A$  and  $\angle B$  are supplementary angles and  $\angle A$  is three times as large as  $\angle B$ , find the measures of  $\angle A$  and  $\angle B$ .

14. Higher Order Thinking The measure of  $\angle DBE$  is  $(0.1x - 22)^\circ$  and the measure of  $\angle CBE$  is  $(0.3x - 54)^\circ$ . Find the value of  $x$ .



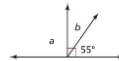
15. Reasoning  $\angle 1$  and an angle that measures  $50^\circ$  are supplementary. Another angle that measures  $50^\circ$  and  $\angle 3$  are supplementary. Show that  $m\angle 1$  and  $m\angle 3$  are equal.

**Assessment Practice**

16. Using the diagram at the right, Martin incorrectly writes  $m\angle b = 125^\circ$ .

PART A

Find the correct measure of  $\angle b$ .



PART B

What mistake did he likely make?

- Ⓐ He subtracted  $55^\circ$  from  $90^\circ$  instead of  $180^\circ$ .
- Ⓑ He subtracted  $55^\circ$  from  $180^\circ$  instead of  $90^\circ$ .
- Ⓒ He added  $55^\circ$  to  $180^\circ$  instead of  $90^\circ$ .
- Ⓓ He added  $55^\circ$  to  $90^\circ$  instead of  $180^\circ$ .

17. In the diagram at the right,  $m\angle 1 = (133 - y)^\circ$ ,  $m\angle 2 = 22^\circ$ , and  $m\angle 3 = (x + 48)^\circ$ . Find the values of  $x$  and  $y$ .

Handwritten solution for problem 17:

$$(90 + 22) + x + 48 = 180$$

$$x + 160 = 180$$

$$-160 = -160$$

$$x = 20$$

$$133 - y = 90$$

$$-y = -43$$

$$y = 43$$

