

Solve & Discuss It!

Kane has 4 pieces of wood available to build a triangle-shaped garden. Which pieces of wood can he use?

2 feet 3 feet
4 feet
5 feet

Make Sense and Persevere Try all possible combinations of three pieces of wood.

Lesson 8-3

Draw Triangles with Given Conditions

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I can...
draw triangles when given information about their side lengths and angle measures.

Focus on math practices

Use Structure Are there any combinations of three pieces of wood that will not create a triangle? Explain.

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Essential Question How can you determine when it is possible to draw a triangle given certain conditions?

EXAMPLE 1 **Draw Triangles with Given Side Lengths**

Students in woodshop class are measuring and cutting out a triangular base for a corner shelf, with sides measuring 6 inches, 8 inches, and 10 inches. How can you determine if all the students will cut out the same triangle? Explain.

Look for Relationships Does the orientation of a triangle change its shape?

Use geometry software to draw and compare triangles with the given side lengths.

You can "turn" the triangles so they fit exactly on each other.

Triangles with the same side lengths are the same shape and size, no matter how they are positioned. So, all the students will cut out the same triangle.

Try It!

How many unique triangles can be drawn with given side lengths of 8 inches, 10.3 inches, and 13 inches?

unique triangle(s) can be drawn with the given side lengths.

Convince Me! When two sides of a triangle are switched, why is it still considered the same triangle?

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EXAMPLE 2 Determine Possible Side Lengths of Triangles

Steve gathers three pieces of wood from the scrap pile in woodshop class.

a. Can Steve make a triangle with these three wood pieces? Explain.

No. The 3-ft and 4-ft wood pieces are not long enough to form a triangle. $3 + 4 < 8$

b. **Generalize** What can you conclude about the lengths that make a triangle possible?

The sum of the lengths of the two shortest sides must be greater than the length of the third side in order to form a triangle.

EXAMPLE 3 Draw a Triangle with a Combination of Given Side Lengths and Angle Measures

Can more than one triangle be drawn with the following conditions?

a. side lengths of 5 inches and 6 inches with an angle of 45°

Draw a 45° angle with line segments of 5 inches and 6 inches.

Connect the sides by drawing the third side.

Generalize Is there more than one way to connect the two ends of the given sides?

Only one triangle can be drawn with the given measures.

b. a side length of 6 inches with angles at each end measuring 40° and 60°

Draw a 6-inch line segment and rays that form the 40° and 60° angles.

Extend the rays until they intersect.

Generalize Is there more than one way to draw the two other sides?

Only one triangle can be drawn with the given measures.

Try It!

a. Write three side lengths that will form a triangle. Write three side lengths that will NOT form a triangle.

b. Can a triangle be drawn with a side length of 3 inches and angles at each end measuring 90° and 89° ? Explain.

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EXAMPLE 4 Draw a Triangle with Two Given Side Lengths and a Nonincluded Angle Measure

Can more than one triangle be drawn using two side lengths of 6 units and 9 units, and a 40° angle that is not formed by their intersection?

Draw $\triangle ABC$ with side lengths 6 units and 9 units, and a nonincluded angle of 40° .

Swing side AB left to create an obtuse triangle, keeping $m\angle C$ at 40° .

The new triangle still has the given side lengths and angle measure, but it is a different triangle.

So, more than one triangle can be made with the given measures.

EXAMPLE 5 Draw a Triangle with Three Given Angle Measures

Is there a unique triangle with angle measures of 30° , 60° , and 90° ?

Draw a triangle with the given angle measures. Notice that side lengths are not required.

Enlarge and reduce your drawing, keeping the angle measures the same but changing the side lengths to proportional measurements.

Generalize Is there more than one way to draw a triangle with three given angles?

Many different triangles can be drawn with the given angle measures.

So, there is not one unique triangle with angle measures of 30° , 60° , and 90° .

Try It!

Can more than one triangle be drawn with two side lengths of 6 inches and a nonincluded angle of 60° ? Explain.

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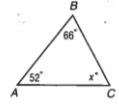
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10-4 Study Guide and Intervention
Triangles

$$\frac{360}{2} = 180^\circ$$

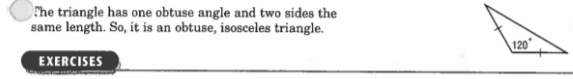
A triangle is a figure with three sides and three angles. The sum of the measures of the angles of a triangle is 180°. You can use this to find a missing angle measure in a triangle.

EXAMPLE 1 Find the value of x in $\triangle ABC$.
 $x + 66 + 52 = 180$ The sum of the measures is 180.
 $x + 118 = 180$ Simplify.
 $-118 \quad -118$ Subtract 118 from each side.
 $x = 62$
 The missing angle is 62° .



Triangles can be classified by the measures of their angles. An acute triangle has three acute angles. An obtuse triangle has one obtuse angle. A right triangle has one right angle, $= 90^\circ$.
 Triangles can also be classified by the lengths of their sides. Sides that are the same length are congruent segments and are often marked by tick marks. In a scalene triangle, all sides have different lengths. An isosceles triangle has at least two congruent sides. An equilateral triangle has all three sides congruent.
 4 same size, shape

EXAMPLE 2 Classify the triangle by its angles and by its sides.



EXERCISES
 Find the missing measure in each triangle. Then classify the triangle as acute, right, or obtuse.

1. $82^\circ, 75^\circ, x^\circ$

$$\begin{array}{r} 82 \\ +73 \\ \hline 157 \end{array}$$

$$\begin{array}{r} 180 \\ -157 \\ \hline 23 \end{array}$$
 $x = 23^\circ$ acute Δ

2. $x^\circ, 47^\circ, 43^\circ$

$$\begin{array}{r} 47 \\ +43 \\ \hline 90 \end{array}$$

$$\begin{array}{r} 180 \\ -90 \\ \hline 90 \end{array}$$
 $x = 90^\circ$ right Δ

3. $45^\circ, 40^\circ, x^\circ$

$$\begin{array}{r} 45 \\ +40 \\ \hline 85 \end{array}$$

$$\begin{array}{r} 180 \\ -85 \\ \hline 95 \end{array}$$
 $x = 95^\circ$ obtuse Δ

Classify each triangle by its angles and by its sides.

4. $60^\circ, 90^\circ, 70^\circ$
 acute, scalene

5. $90^\circ, 45^\circ, 45^\circ$
 right, isosceles

6. $40^\circ, 30^\circ, 110^\circ$
 obtuse, scalene

10-4 Practice: Skills
Triangles

Find the missing measure in each triangle. Then classify the triangle as acute, right, or obtuse.

1. $61^\circ, x^\circ, 84^\circ$

2. $36^\circ, 24^\circ, x^\circ$

3. $x^\circ, 49^\circ, 90^\circ$

4. $x^\circ, 38^\circ, 38^\circ$

5. $65^\circ, x^\circ, 90^\circ$

6. $71^\circ, x^\circ, 45^\circ$

7. $57^\circ, 51^\circ, x^\circ$

8. $x^\circ, 126^\circ, 22^\circ$

9. $x^\circ, 50^\circ, 90^\circ$

Classify each triangle by its angles and by its sides.

10. $37^\circ, 60^\circ, 60^\circ$

11. $114^\circ, 40^\circ, 90^\circ$

12. $90^\circ, 40^\circ, 90^\circ$

13. $60^\circ, 60^\circ, 60^\circ$

14. $90^\circ, 40^\circ, 90^\circ$

15. $121^\circ, 19^\circ, 40^\circ$

16. $98^\circ, 46^\circ, 90^\circ$

17. $82^\circ, 46^\circ, 52^\circ$

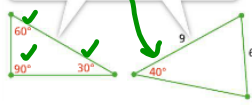
18. $27^\circ, 90^\circ, 90^\circ$

Lesson 10-4

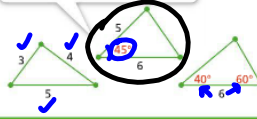
KEY CONCEPT

You can analyze given conditions of side lengths and angle measures to determine whether one unique triangle, more than one unique triangle, or no triangle can be drawn.

There is **more than one** possible triangle given these cases: **all three angles**, or **two sides and a nonincluded angle**.



There is **one unique triangle** given these cases: **all three sides**, **two sides and an included angle**, or **two angles and an included side**.



Do You Understand?

- Essential Question** How can you determine when it is possible to draw a triangle given certain conditions?
- Look for Relationships** What is the relationship between all triangles that can be drawn given the same three angle measures?
- Why can there be only one way to draw a triangle if two sides and an included angle are given?

Do You Know How?

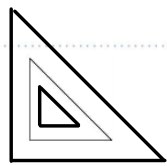
- How many triangles can be drawn with side lengths 4 centimeters, 4.5 centimeters, and 9 centimeters? Explain.
- Can more than one triangle be drawn with side lengths of 5 inches and 7 inches and an included angle with a measure of 50° ? Explain.
- Sketch two different triangles that have angle measures of 45° , 45° , and 90° .

There is only one unique triangle when given 2 side lengths and one included angle.

Name: _____

Practice & Problem Solving

- Draw two different triangles with angle measurements 90° , 35° , and 55° .
- If you form a triangle from three given side lengths, will you always get one triangle or more than one triangle?
- How can you make different-looking triangles given two of the angle measures and the included side lengths?
- If you form a triangle from two given angle measures and the length of the included side, will you always get one triangle or will you get more than one triangle?
- How can you make different triangles with the same angle measures?
- Given two side lengths of 15 units and 9.5 units, with a nonincluded angle of 75° , can you draw no triangles, only one triangle, or more than one triangle?
- A student was asked to form different triangles with angle measures of 90° , 30° , and 60° . She incorrectly said this triangle is the only triangle with angle measures of 90° , 30° , and 60° . What mistake might she have made?

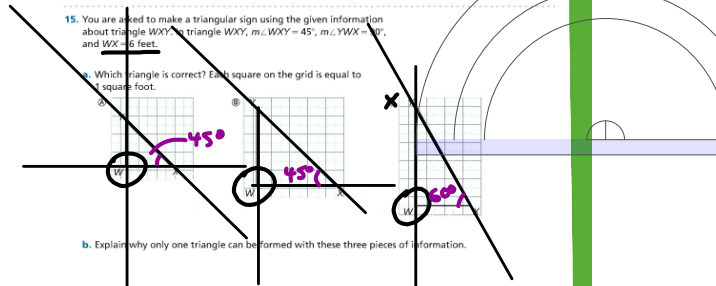


14. In triangle QRS , $m\angle QSR = 100^\circ$, $m\angle SQR = 45^\circ$, and $QR = 4$ units. In triangle XYZ , $m\angle XYZ = 100^\circ$, $m\angle ZXY = 45^\circ$, and $XY = 4$ units. Are triangles QRS and XYZ the same? Explain.

use graph paper

15. You are asked to make a triangular sign using the given information about triangle WXY : In triangle WXY , $m\angle WXY = 45^\circ$, $m\angle YWX = 10^\circ$, and $WX = 6$ feet.

a. Which triangle is correct? Each square on the grid is equal to 1 square foot.



b. Explain why only one triangle can be formed with these three pieces of information.

16. **Look for Relationships** Two different triangles have side lengths of 13 and 16 units and a nonincluded angle of 50° . Explain how the triangles are different.

17. **Higher Order Thinking** Two triangles have side lengths of 12 units and 15 units and the non-included angle of 45° . Draw two different triangles with these conditions.

18. For triangle RST , RS is 12 centimeters, ST is 16 centimeters, and RT is 19 centimeters. How many triangles can be drawn with the given side lengths?

19. A triangle has two side lengths of 8.5 centimeters and 9.5 centimeters. What is a possible length for the third side? Explain why this is a possible length.

* sum of any 2 sides of Δ has to be larger than the 3rd side

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9.5 (largest)

$$8.5 + ? > 9.5$$

$$8.5 + \textcircled{2} = 10.5 > 9.5$$

$$8.5 + 3 = 11.5 > 9.5$$

20. Can a triangle be formed with side lengths of 4, 5, and 7 units?

Assessment Practice

21. Which of the following combinations of side lengths would form a triangle? Select all that apply. *

- 7 in., 10 in., 2.5 in.
- 4.5 ft, 8 ft, 5 ft
- 5 yd, 11 yd, 5 yd
- 12 in., 5 in., 9.5 in.
- 7 m, 7 m, 9 m
- 6 ft, 16 ft, 9 ft

* sum of 2 shorter sides > longest side

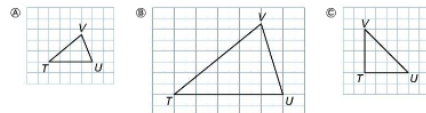
$$5 + 9.5 = 14.5 > 12 \text{ Yes}$$

$$6 + 9 = 15 < 16 \text{ No}$$

22. Choose the triangle that matches the given conditions. One grid square is one square unit.

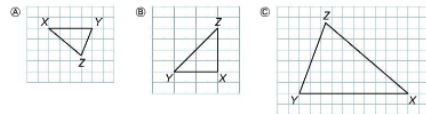
PART A

Triangle TUV , where $m\angle TVU = 70^\circ$, $m\angle VTU = 40^\circ$, and $TU = 4$ units



PART B

Triangle XYZ , where $m\angle XZY = 70^\circ$, $m\angle ZXY = 40^\circ$, and $XY = 4$ units



PART C

Are triangles TUV and XYZ the same? Explain.

