


### Explore It!

The Great Karlo called twins Jasmine and James onto the stage.

*Jasmine, multiply your age by 3 and add 6. Then multiply this sum by 2. James, multiply your age by 2 and add 4. Then multiply this sum by 3. I predict you will both get the same number!*



**Lesson 7-4**  
Equations with No Solutions or Infinitely Many Solutions

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**I can...**  
determine the number of solutions an equation has.

**A.** Write expressions to represent Great Karlo's instructions to each twin.

**B.** Choose 4 whole numbers for the twins' age and test each expression. Make a table to show the numbers you tried and the results.

**C.** What do you notice about your results?

**Focus on math practices**  
**Make Sense and Persevere** Choose three more values and use them to evaluate each expression. What do you notice? Do you think this is true for all values? Explain.

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**Essential Question** Will a one-variable equation always have only one solution?

**EXAMPLE 1** Solve an Equation with Infinitely Many Solutions

For what values of  $x$  will the rectangle and triangle have the same perimeter?

$x$   
 $2x + 3$

$2x + 2$   
 $\triangle$   
 $2x + 2$

**Model with Math** How can you use bar diagrams to represent the equal perimeters?

**ONE WAY** Draw bar diagrams to represent the perimeters. Then decompose and reorder the bar diagrams to solve for  $x$ .

$2x + 3$	$2x + 3$	$x$
$2x + 2$	$2x + 2$	$2x + 2$

$2x + 3$	$2x + 3$	$2x$
$x$	$x$	$x$
$x$	$x$	$x$
$x$	$x$	$x$
$2x + 2$	$2x + 2$	$2x + 2$

$x$	$x$	$x$	$x$	$3$	$3$
$x$	$x$	$x$	$x$	$2$	$2$

$6x = 6x$        $6 = 6$

The expressions  $6x = 6x$  and  $6 = 6$  are true for any value of  $x$ . This equation has infinitely many solutions.

**ANOTHER WAY** Write an equation to represent the equal perimeters. Then use inverse operations and properties of equality to solve.

$$2x + 3 + 2x + 3 + x + x = 2x + 2 + 2x + 2 + 2x + 2$$

$$6x + 6 = 6x + 6$$

$$6x - 6x + 6 = 6x - 6x + 6$$

$$6 = 6$$

For what values of  $x$  will  $6x + 6 = 6x + 6$ ?

Because  $6 = 6$  is always true, all values of  $x$  will make the equation true.

This equation has infinitely many solutions.

**Try It!**

How many solutions does the equation  $3x + 15 = 2x + 10 + x + 5$  have? The equation has \_\_\_\_\_ solutions.

**Convince Me!** If the value of  $x$  is negative, would the equation still be true? Explain.

$$3x + 15 = 2x + 10 + x + 5$$

$$3x + 15 = \square x + \square$$

$$3x - \square + 15 = 3x - \square + 15$$

$$\square = \square$$

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**EXAMPLE 2** Solve an Equation with One Solution

Anna and Lee played soccer for the same number of hours one week. How many hours did Lee play on Sunday?

**ONE WAY** Use bar diagrams to solve.

Anna's Schedule	$x + 1.2$	$x + 1.2$	$x + 1.2$	$x + 1.2$
Lee's Schedule	$x$	2.5	$2x$	4.5

$x$	$x$	$x$	$x$	4.8
$x$	$x$	$x$	2.2	4.8

$x = 2.2$

**ANOTHER WAY** Write and solve an equation.

$$4(x + 1.2) = x + 2.5 + 2x + 4.5$$

$$4x + 4.8 = x + 2.5 + 2x + 4.5$$

$$4x + 4.8 = 3x + 7.0$$

$$4x - 3x + 4.8 = 3x - 3x + 7.0$$

$$x + 4.8 - 4.8 = 7.0 - 4.8$$

$$x = 2.2$$

This equation has one solution,  $x = 2.2$ .

Lee played soccer for 2.2 hours on Sunday.

**EXAMPLE 3** Solve an Equation with No Solution

Gil makes 3 bracelets and Mika makes 2 bracelets. They both use the same number of string colors. How many colors should they use to make the same amount of money?

Write an equation to represent this situation. Then solve.

Let  $x$  = the number of string colors.

$$3(2x + 5) = 2(3x + 3)$$

$$6x + 15 = 6x + 6$$

$$6x - 6x + 15 = 6x - 6x + 6$$

$$15 = 6$$

Because 15 can never equal 6, this equation has no solution.

Because  $15 \neq 6$ , there is no number of string colors that results in Gil and Mika making the same amount of money.

**Try It!**

How many solutions does the equation  $4x + 8 = 0.1x + 3 + 3.9x$  have? Explain.

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**EXAMPLE 4** Determine the Number of Solutions by Inspection

How can you determine the number of solutions each equation has without solving?

**a.**  $x + 3 + 7 = 2x - 10 - x$

$$x + 10 \neq x - 10$$

You can combine like terms mentally. The equivalent expressions  $x + 10$  and  $x - 10$  are not true for any values of  $x$ .

The equation  $x + 3 + 7 = 2x - 10 - x$  has no solutions.

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**b.**  $3(x + 4) = 3x + 12$

$$3x + 12 = 3x + 12$$

You can apply the Distributive Property on the left side of the equation mentally. It is easy to see that the equivalent equation  $3x + 12 = 3x + 12$  is true for all values of  $x$ .

The equation  $3(x + 4) = 3x + 12$  has infinitely many solutions.

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**c.**  $5x + 8 = 2x - 1$

$$3x = -9$$

Notice that the coefficients of the variable terms are different. When like terms are collected and combined, the result will be a unique value of  $x$ .

The equation  $5x + 8 = 2x - 1$  has one solution.

**Try It!**

Determine the number of solutions each equation has without solving. Explain your reasoning.

**a.**  $3x + 1.5 = 2.5x + 4.7$     **b.**  $3(x + 2) = 3x - 6$     **c.**  $9x - 4 = 5x - 4 + 4x$

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**KEY CONCEPT**

A one-variable equation has **infinitely many solutions** when solving results in a true statement, such as  $2 = 2$ .

A one-variable equation has **one solution** when solving results in one value for the variable, such as  $x = 2$ .

A one-variable equation has **no solution** when solving results in an untrue statement, such as  $2 = 3$ .

$x \in \mathbb{R}$

$2 \neq 3$

**Do You Understand?**

- 1. Reasoning** Will a one-variable equation always have only one solution?
- 2. Use Structure** Kaylee writes the equation  $6x + 12 = 2(3x + 6)$ . Can you find the number of solutions this equation has without solving for  $x$ ? Explain.
- 3. Construct Arguments** The height of an experimental plant after  $x$  days can be represented by the formula  $3(4x + 2)$ . The height of a second plant can be represented by the formula  $6(2x + 2)$ . Is it possible that the two plants will ever be the same height? Explain.

**Do You Know How?**

- How many solutions does the equation  $7x + 3x - 8 = 10x - 8$  have? Explain.
- How many solutions does the equation  $3(2.4x + 4) = 4.1x + 7 + 3.1x$  have? Explain.

$$\begin{aligned}
 7x + 3x - 8 &= 10x - 8 \\
 + -7x & \quad + -7x \\
 \hline
 0x - 8 &= 0x - 8 \\
 -8 &= -8
 \end{aligned}$$

**infinitely many solutions**  
or  
 $x \in \mathbb{R}$

- Todd and Agnes are making desserts. Todd buys peaches and a carton of vanilla yogurt. Agnes buys apples and a jar of honey. They bought the **same number of pieces of fruit**. Is there a situation in which they pay the same amount for their purchases? Explain.



$$\begin{aligned}
 1.25x + 4 &= 1x + 6 \\
 + -1.00x & \quad + -1x \\
 \hline
 0.25x + 4 &= 0x + 6 \\
 + -4 & \quad + -4 \\
 \hline
 0.25x &= 2 \\
 \frac{0.25x}{0.25} &= \frac{2}{0.25}
 \end{aligned}$$

$$\begin{aligned}
 x &= 0.25 \sqrt{200} \\
 x &= 8
 \end{aligned}$$

Name: \_\_\_\_\_

**Practice & Problem Solving**

**Leveled Practice** In 7 and 8, complete the equations to find the number of solutions.

- Classify the equation  $33x + 99 = 33x - 99$  as having one solution, no solution, or infinitely many solutions.

$$\begin{aligned}
 33x + 99 &= 33x - 99 \\
 33x - 33x + 99 &= 33x - 33x - 99 \\
 99 &\neq -99
 \end{aligned}$$

Since 99 is **NOT** equal to -99, the equation has **NO** solution(s).

$$\begin{aligned}
 33x + 99 &= 33x - 99 \\
 + -33x & \quad + -33x \\
 \hline
 99 &\neq -99 \\
 \emptyset
 \end{aligned}$$

- Solve  $4(4x + 3) = 19x + 9 - 3x + 3$ . Does the equation have one solution, no solution, or infinitely many solutions?

$$\begin{aligned}
 4(4x + 3) &= 19x + 9 - 3x + 3 \\
 4 \cdot \square + 4 \cdot \square &= 19x + 9 - 3x + 3 \\
 16x + 12 &= \square + \square \\
 16x - \square + 12 &= 16x - \square + 12 \\
 12 &= 12
 \end{aligned}$$

Since 12 is **equal** to 12, the equation has **infinitely many** solution(s).

- Generalize** What does it mean if an equation is equivalent to  $0 = 0$ ? Explain.

- Solve  $4x + x + 4 = 8x - 3x + 4$ . Does the equation have one solution, no solution, or infinitely many solutions? If one solution, write the solution. Explain.

$$\begin{aligned}
 4x + x + 4 &= 8x - 3x + 4 \\
 \cancel{4x} + \cancel{x} + 4 &= \cancel{8x} - \cancel{3x} + 4 \\
 + -4 & \quad + -4 \\
 \hline
 4 &= 4
 \end{aligned}$$

- Reasoning** Two rival dry cleaners both advertise their prices. Let  $x$  equal the number of items dry cleaned. Store A's prices are represented by the expression  $15x - 2$ . Store B's prices are represented by the expression  $3(5x + 7)$ . When do the two stores charge the same rate? Explain.

**infinitely many solutions**  
 $x \in \mathbb{R}$

12. **Reasoning** How is solving an equation with no solution similar to solving an equation that has an infinite number of solutions?

13. Solve  $0.9x + 5.1x - 7 = 2(2.5x - 3)$ . How many solutions does the equation have?

14. **Critique Reasoning** Your friend solved the equation  $4x + 12x - 6 = 4(4x + 7)$  and got  $x = 34$ . What error did your friend make? What is the correct solution?

Handwritten work for problem 14:

$$4x + 12x - 6 = 4(4x + 7)$$

$$16x - 6 = 16x + 28$$

$$16x - 16x = 28 + 6$$

$$0 = 34$$

The student circled the  $16x$  terms and crossed out the  $-6$  and  $+28$  terms, incorrectly concluding  $x = 34$ .

$6 \neq 28$   $\emptyset$   $83 + 5x = 2 + 5x$  no solution

15. Solve  $49x + 9 = 49x + 83$ .

- a. Does the equation have one solution, no solution, or infinitely many solutions?  
 b. Write two equations in one variable that have the same number of solutions as this equation.

Handwritten work for problem 15:

$$49x + 9 = 49x + 83$$

$$-49x + 9 = -49x + 83$$

$$9 \neq 83$$

The student concludes: "The equation  $0x + 9 = 0x + 83$  has no solution." and provides two example equations:  $9 \neq 83$  and  $\emptyset$  (no solution).

16. Classify the equation  $6(x + 2) = 5(x + 7)$  as having one solution, no solution, or infinitely many solutions.

17. Solve  $6x + 14x + 5 = 5(4x + 1)$ . Write a word problem that this equation, or any of its equivalent forms, represents.

18. Classify the equation  $170x - 1,000 = 30(5x - 30)$  as having one solution, no solution, or infinitely many solutions.

19. **Higher Order Thinking** Write one equation that has one solution, one equation that has no solution, and one equation that has infinitely many solutions.

20. Solve  $4(4x - 2) + 1 = 16x - 7$ .

21. Solve  $6x + 26x - 10 = 8(4x + 10)$ .

22. Classify the equation  $64x - 16 = 16(4x - 1)$  as having one solution, no solution, or infinitely many solutions.

23. Classify the equation  $5(2x + 3) = 3(3x + 12)$  as having one solution, no solution, or infinitely many solutions.

**Assessment Practice**

24. Which of the following best describes the solution to the equation  $4(2x + 3) = 16x + 12 - 8x$ ?
- A The equation has one solution.
  - B The equation has infinitely many solutions.
  - C The equation has no solution.

25. Which of the following statements are true about the equation  $10x + 45x - 13 = 11(5x + 6)$ ? Select all that apply.
- The operations that can be used to solve the equation are addition and multiplication.
  - The operations that can be used to solve the equation are multiplication and division.
  - The equation has infinitely many solutions.
  - The equation has a solution of  $x = 53$ .
  - The equation has no solution.

