Lesson 7-2
Understand Theoretical Probability

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I can...
determine the theoretical probability of an event.

Solve & Discuss It!

Betty and Carl will conduct an experiment. They will flip a coin 100 times and record the result of each flip. What should they expect the results of their experiment to be? Justify your answer.

Focus on math practices

Look for Relationships How would their expected results change if Betty and Carl flipped a coin 500 times?

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Essential Question How can the probability of an event help make predictions?

EXAMPLE 1 Use Theoretical Probability to Make Predictions

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Talia and Yoshi design a game for the school fair. Contestants spin the pointer and win a prize if it lands on either of the two red sections. How can Talia and Yoshi determine how many people are likely to be winners if 500 people play their game?

Model with Math How can you use probability to predict the number of winners?

STEP 1 Determine the total number of possible outcomes from one spin of the pointer.

There are 16 sections that are all the same size. There are 2 sections of each color.

The pointer could land on any of the 16 sections, so there are 16 possible equally likely outcomes.

STEP 2 Because you know all the possible outcomes, you can find the **theoretical probability** of an event, such as the pointer landing on red.

P represents theoretical probability. An **event** is a single outcome or group of outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

The event is *landing on red.*

$$P(\text{red}) = \frac{\text{number of red sections}}{\text{total number of sections}} = \frac{2}{16} = \frac{1}{8}$$

The theoretical probability that a contestant will win this game is $\frac{1}{8}$ or 12.5%.

STEP 3 Use proportional reasoning to predict the number of likely winners.

w represents the number of likely winners.

$$\frac{1}{8} = \frac{w}{500}$$

$$\frac{1}{8} \cdot 500 = \frac{w}{500} \cdot 500$$

$$62.5 = w$$

Of the 500 contestants, about 62 are likely to be winners.

Try It!

If Talia and Yoshi redesign their spinner to have 14 sections instead of 16 sections, would they likely have more or fewer winners? Explain why.

Convince Me! If there are always 2 red sections, how does the number of total sections in the spinner relate to the theoretical probability of winning this game?

EXAMPLE 2 Determine Theoretical Probability

Archie plays a word board game. He places 98 lettered tiles and 2 blank tiles in a bag. Players will draw tiles from the bag one at a time without looking.

What is the probability that the first tile drawn will be a blank tile? Labeled with a letter? A vowel? A consonant?

$P(\text{blank}) = \frac{2}{100} = \frac{1}{50}$ $P(\text{vowel}) = \frac{42}{100} = \frac{21}{50}$

$P(\text{letter}) = \frac{98}{100} = \frac{49}{50}$ $P(\text{consonant}) = \frac{56}{100} = \frac{28}{50}$

Look for Relationships How are the probabilities related to each other?

A	A	A	A	A	A	A	A	A	E
E	E	E	E	E	E	E	E	E	E
E	I	I	I	I	I	I	I	I	I
O	O	O	O	O	O	O	O	U	U
U	U	B	B	C	C	D	D	D	D
F	F	G	G	G	H	H	J	K	L
L	L	L	M	M	N	N	N	N	N
N	P	P	Q	R	R	R	R	R	R
S	S	S	S	T	T	T	T	T	T
V	V	W	W	X	Y	Y	Z		

EXAMPLE 3 Use Theoretical Probability to Make More Predictions

Joaquin also designs a game for the school fair. Contestants roll two number cubes at the same time. If the sum of the numbers on the two cubes is 7, the player wins. About how many winners should Joaquin expect if 500 contestants play his game?

STEP 1 Find all possible outcomes and all winning outcomes.

2nd number cube

1st number cube	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are 6 ways to win out of a total of 36 possible combinations of rolls.

STEP 2 Find the theoretical probability of rolling a sum of 7.

$P(\text{sum of 7}) = \frac{6}{36} = \frac{1}{6}$

STEP 3 Use proportional reasoning to predict the number of winners, w.

$\frac{1}{6} = \frac{w}{500}$

$\frac{1}{6} \cdot 500 = \frac{w}{500} \cdot 500$

$83.\overline{3} = w$

Of 500 contestants, Joaquin should expect about 83 winners.

Try It!

Joaquin wants to reduce the number of winners so he does not have to prepare as many prizes. Choose another sum he could use as a winning sum, and predict the number of winners if 500 people play his game.

KEY CONCEPT

$P(\text{want}) = \frac{\text{want}}{\text{total}}$ → theory

You can determine the theoretical probability of an event, $P(\text{event})$, if you know all the possible outcomes and they are equally likely.

$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$

You can use theoretical probability and proportional reasoning to make predictions, such as in a game situation.

$\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}} = \frac{\text{number of winning outcomes}}{\text{total number of possible outcomes}}$

Do You Understand?

1. **Essential Question** How can the probability of an event help make predictions?

2. **Construct Arguments** A game board has a spinner with 10 equal-sized sections, of which 4 are green, 3 are blue, 2 are yellow, and 1 is red. What is the sum of the probabilities of the pointer landing in the green, blue, yellow, and red sections? Explain.

3. **Reasoning** What does it mean that there is an equal theoretical probability of each outcome? Explain.

Do You Know How?

1, 2, 3, 4, 5, 6

In 4–6, Monique rolls a six-sided number cube labeled 1 to 6.

4. Find $P(\text{rolling a 4})$.

$P(4) = \frac{1}{6} = \frac{P}{100}$

$1 \div 6 = 0.166\ldots$ → $\approx 16.66\%$ → $\approx 17\%$

5. Find $P(\text{rolling an odd number})$.

1, 3, 5

$P(\text{odd}) = \frac{3}{6} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100}$

$= 50\%$

6. If Monique rolls the number cube 12 times, how many times would she expect a number greater than 4 to be rolled?

Name: _____

Practice & Problem Solving

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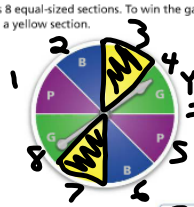
Leveled Practice In 7–9, complete each statement.

7. A spinner has 8 equal-sized sections. To win the game, the pointer must land on a yellow section.



Yellow / Total = 2/8 = 1/4

P(yellow) = favorable outcomes / total number of possible outcomes = 2/8 = 1/4 = 0.25 = 25 %

8. Natalie is playing a game using a fair coin. Contestants win the game if the fair coin lands tails up.

The theoretical probability that the coin will land tails up is _____.

If 250 contestants play the game, about _____ of them are expected to win.

9. In a different game, the probability of correctly guessing which of 5 boxes contains a tennis ball is $\frac{1}{5}$. About how many winners would be expected if 60 contestants play the game?

$\frac{1}{5} = \frac{x}{60}$ ball / total $\frac{1 \times 12}{5 \times 12} = \frac{x}{60}$

$x =$ 12 winners

10. Make Sense and Persevere A 12-sided solid has equal-sized faces numbered 1 to 12.

a. Find P(number greater than 10).

b. Find P(number less than 5).

c. If the 12-sided solid is rolled 200 times, how many times would you expect either a 4, 6, or 9 to be rolled?

* see p.363 ex 2 table

11. Tamara finds the sum of two number cubes rolled at the same time. The chart below shows all possible sums from the 36 possible combinations when rolling two number cubes. How many times should Tamara expect the sum of the two cubes to be equal to 5 if she rolls the two number cubes 180 times?

Sum	2	3	4	5	6	7	8	9	10	11	12
Possible Combinations	1	2	3	4	5	6	5	4	3	2	1

12. Higher Order Thinking A store is giving every customer who enters the store a scratch-off card labeled with numbers from 1 to 10. It is equally likely that any of the numbers from 1 to 10 will be labeled on a given card. If the card is an even number, the customer gets a 15% discount on a purchase. If the card is an odd number greater than 6, the customer gets a 30% discount. Otherwise, the discount is 20%.

a. What is the probability for each discount?

15% discount: _____ 20% discount: _____

30% discount: _____

b. The store manager gives out 300 scratch-off cards. Which discount will the greatest number of customers likely receive? Explain.

Assessment Practice

13. A city traffic light cycles through green, yellow, and red in 60 seconds. It is yellow for 3 seconds. Which term best describes the likelihood of a car coming to the light when it is yellow?

$\frac{3}{60} \rightarrow \frac{1 \text{ yellow sec.}}{20 \text{ total sec.}}$

Ⓐ Impossible

● Unlikely

Ⓒ Just as likely as not likely

Ⓓ Likely

Ⓔ Certain

14. Based on the records for the past several seasons, a sports fan believes the probability that the red team will win a given game is $\frac{3}{5}$. The fan also believes the probability that the blue team wins a given game is $\frac{11}{20}$. As compared to the blue team, how many more games should the fan expect the red team to win during a 180-game season? Explain.