

Solve & Discuss It!

Weight is a measure of force affected by gravity. The Moon's gravity is less than Earth's gravity, so objects weigh less on the Moon than on Earth.

Using the information provided, how much do you think a cat will weigh on the Moon? Explain your reasoning.

Lesson 3-3
Understand Proportional Relationships: Equivalent Ratios

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I can...
test for equivalent ratios to decide whether quantities are in a proportional relationship.

Make Sense and Persevere
About how much does a cat weigh on Earth?

Focus on math practices

Generalize How could you find the approximate weight of any object on the Moon? Explain your reasoning.

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Essential Question How are proportional quantities described by equivalent ratios?

INTERACTIVE ANIMATIONS

EXAMPLE 1 Recognize a Proportional Relationship

On Sarah's favorite mobile game, she is awarded game lives when she finds gold coins. How is the number of game lives awarded related to the number of gold coins found? Explain.

Write ratios that relate the gold coins found and the game lives awarded.

$$\frac{3 \text{ coins}}{9 \text{ lives}} = \frac{2}{6}$$

$$\frac{5 \text{ coins}}{15 \text{ lives}} = \frac{1}{3}$$

Make a ratio table to determine whether the ratios are equivalent.

| Coins | 3 | 2 | 5 |
|------------|-----------------------------|-----------------------------|------------------------------|
| Game Lives | 9 | 6 | 15 |
| Coins | $\frac{3}{9} = \frac{1}{3}$ | $\frac{2}{6} = \frac{1}{3}$ | $\frac{5}{15} = \frac{1}{3}$ |
| Game Lives | | | |

Two quantities are in a **proportional relationship** if all of the ratios that relate the quantities are equivalent.

Each ratio $\frac{\text{coins}}{\text{game lives}}$ is equivalent to $\frac{1}{3}$. The number of game lives awarded is proportional to the number of gold coins found.

Try It!

Miles records the time it takes to download a variety of file types. How is the download time related to the file size? Explain.

The ratios for each pair of data are _____, so the download time and the file size are _____.

Convince Me! How can you show that two quantities have a proportional relationship?

| Type of Media | File Size (MB) | Download Time (s) | Download Time / File Size |
|---------------|----------------|-------------------|---------------------------|
| Document | 1.25 | 25 | $\frac{25}{1.25} = 20$ |
| Song | 3.6 | 72 | $\frac{72}{3.6} = 20$ |
| Video | 6.25 | 125 | $\frac{125}{6.25} = 20$ |

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EXAMPLE 2 **Decide Whether Quantities are Proportional**

Is the relationship between the area and the side length of the squares proportional? Explain your reasoning.

STEP 1 Make a table to organize the data. Find the ratio of area to side length for each data pair.

| Side Length (x) | Area (y) | $\frac{\text{Area (y)}}{\text{Side Length (x)}}$ |
|-----------------|----------|--|
| 2 | 4 | $\frac{4}{2} = 2$ |
| 3 | 9 | $\frac{9}{3} = 3$ |
| 4 | 16 | $\frac{16}{4} = 4$ |

STEP 2 Compare the ratios by finding the unit rates.

$\frac{2}{1} \neq \frac{3}{1} \neq \frac{4}{1}$

The ratios are not equivalent so the relationship between the area and side length is NOT proportional.

Try It!

The table at the right shows information about regular hexagons. Is the relationship between the perimeter and the side length of the hexagons proportional? Explain.

| Side Length (x) | Perimeter (y) |
|-----------------|---------------|
| 2 | 12 |
| 3 | 18 |

EXAMPLE 3 **Use Proportions to Solve Problems**

A **proportion** is an equation that represents equal ratios. Use the table at the right. How many times will a hummingbird beat its wings in 60 seconds?

Hummingbird Wing Beats

| Seconds (x) | Wing Beats (y) |
|-------------|----------------|
| 2 | 160 |
| 7 | 560 |
| 10 | 800 |

STEP 1 Verify that the quantities are proportional.

$\frac{160}{2} = \frac{80}{1}$, $\frac{560}{7} = \frac{80}{1}$, $\frac{800}{10} = \frac{80}{1}$

The ratios are equivalent, so the quantities are proportional.

STEP 2 Write and solve a proportion.

$\frac{80}{1} = \frac{y}{60}$

$80 \cdot 60 = \frac{y}{60} \cdot 60$

4,800 = y

A hummingbird beats its wings 4,800 times in 60 seconds.

Try It!

Ginny's favorite cookie recipe requires $\frac{1}{2}$ cups of sugar to make 24 cookies. How much sugar does Ginny need to make 36 of these cookies?

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KEY CONCEPT

Two quantities x and y have a **proportional relationship** if all the ratios $\frac{y}{x}$ for related pairs of x and y are equivalent.

A proportion is an equation that states that two ratios are equivalent.

$\frac{3}{120} = \frac{5}{200}$

| x | y | $\frac{y}{x}$ |
|-----|---|-------------------------|
| 120 | 3 | $\frac{3}{120} = 0.025$ |
| 200 | 5 | $\frac{5}{200} = 0.025$ |

$\frac{1}{2} = \frac{5}{10} = \frac{4}{8}$

$= \frac{2}{4} = \frac{100}{200}$

Do You Understand?

- Essential Question** How are proportional quantities described by equivalent ratios?
- Look for Relationships** How do you know if a relationship between two quantities is NOT proportional?
- Reasoning** If the ratio $\frac{y}{x}$ is the same for all related pairs of x and y , what does that mean about the relationship between x and y ?

Do You Know How?

- Use the table below. Do x and y have a proportional relationship? Explain.

| x | y |
|---|------|
| 2 | 5 |
| 3 | 7.5 |
| 5 | 12.5 |
| 8 | 18 |

- Each triangle is equilateral. Is the relationship between the perimeter and the side length of the equilateral triangles proportional? Explain.



| Side | Perim |
|-------|-------|
| 1 x 3 | 3 |
| 2 x 3 | 6 |
| 3 x 3 | 9 |

- Is the relationship between the number of tickets sold and the number of hours proportional? If so, how many tickets were sold in 8 hours?

| Hours (h) | Tickets Sold (t) |
|-----------|------------------|
| 3 | 240 |
| 5 | 400 |
| 9 | 720 |

Name: _____

Practice & Problem Solving

7. The amount of seed a landscaper uses and the area of lawn covered have a proportional relationship. Complete the table.

| Seed (oz) | 2 | 3 | 4 |
|---------------------------------|---------------------|---------------------|----------------------|
| Area Covered (ft ²) | 50 | 75 | 100 |
| Area Covered Seed | $\frac{50}{2} = 25$ | $\frac{75}{3} = 25$ | $\frac{100}{4} = 25$ |

8. **Construct Arguments** Is the relationship between the number of slices of salami in a sandwich and the number of Calories proportional? Explain.

Calories in a Sandwich

| Slices of Salami | Calories |
|------------------|----------|
| 1 | 66 |
| 2 | 96 |
| 3 | 126 |
| 4 | 156 |

x39
NOT proportional

9. **Look for Relationships** A wholesale club sells eggs by the dozen. Does the table show a proportional relationship between the number of dozens of eggs and the cost? Explain.

Cost of Dozens of Eggs

| Dozen | Cost (\$) |
|-------|-----------|
| 6 | 21 |
| 8 | 28 |
| 10 | 35 |
| 14 | 49 |

10. Does the table show a proportional relationship? If so, what is the value of y when x is 11?

| x | 4 | 5 | 6 | 10 |
|---|----|-----|-----|-------|
| y | 64 | 125 | 216 | 1,000 |

11. Does the table show a proportional relationship? If so, what is the value of y when x is 10?

| x | 5 | 6 | 7 | 8 |
|---|---------------|---|---------------|---------------|
| y | $\frac{2}{3}$ | 2 | $\frac{2}{3}$ | $\frac{2}{3}$ |

12. The height of a building is proportional to the number of floors. The figure shows the height of a building with 9 floors.

a. **Reasoning** Write the ratio of height of the building to the number of floors. Then find the unit rate, and explain what it means in this situation.



$$\frac{y}{x} = \frac{360 \div 4}{160 \div 4} = \frac{9}{4} = 2.25$$

13. **Higher Order Thinking** Do the two tables show the same proportional relationship between x and y? Explain how you know.

| x | 160 | 500 | 1,200 |
|---|-----|-------|-------|
| y | 360 | 1,125 | 2,700 |

$$\frac{4.5}{2} = 2 \frac{2.25}{1} = 4.50$$

$$\frac{1125 \div 25}{500 \div 25} = \frac{45 \div 5}{20 \div 5} = \frac{9}{4}$$

Assessment Practice

14. The table shows the number of cell phone towers a company will build as the number of its customers increases.

PART A
Is the relationship between number of towers and number of customers proportional? Explain.

PART B
If there are 576 towers, how many customers does the company have? Write a proportion you can use to solve.

Cell Phone Towers

| Customers (thousands) | Towers |
|-----------------------|--------|
| 5.25 | 252 |
| 6.25 | 300 |
| 7.25 | 348 |
| 9.25 | 444 |

$$\frac{252}{5250} = 0.048$$

$$\frac{300}{6250} = 0.048$$

15. Which of the following statements about the table is true? Select all that apply.

- The table shows a proportional relationship.
- When x is 20, y is 2.55.
- All the ratios $\frac{y}{x}$ for related pairs of x and y are equivalent to 8.
- The unit rate of $\frac{y}{x}$ for related pairs of x and y is $\frac{1}{8}$.

| x | 12 | 18 | 22 | 26 |
|---|-----|------|------|------|
| y | 1.5 | 2.25 | 2.75 | 3.25 |

