

Explain It!

Sofia wrote a decimal as a fraction. Her classmate Nora says that her method and answer are not correct. Sofia disagrees and says that this is the method she learned.

$$0.1211211121112... = x$$


$$x = 0.12$$

$$100 \cdot x = 100 \cdot 0.12$$

$$100x = 12.12$$

$$99x = 12$$

$$x = \frac{12}{99}$$



Lesson 2-2

Understand Irrational Numbers

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I can...
Identify a number that is irrational.

A. Construct Arguments Is Nora or Sofia correct? Explain your reasoning.

B. Use Structure What is another nonterminating decimal number that can not be written as a fraction.

Focus on math practices

Construct Arguments Is 0.1211211121112... a rational number? Explain.

Essential Question

How is an irrational number different from a rational number?

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EXAMPLE 1 Identify Irrational Numbers

The Venn diagram shows the relationships among rational numbers.

How would you classify the number 0.24758326... ?

Reasoning How can you use the definition of each number set to classify numbers?

0.24758326...

The decimal expansion does not terminate or repeat, so it cannot be written as a ratio of two integers.

The number 0.24758326... is not a rational number.

Numbers that are not rational are called *irrational*. An **irrational number** is a number that cannot be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Rational Numbers
$\frac{4}{5}$
0.75
31.8

Integers
-5
$-\frac{16}{4}$
-1,000

Whole Numbers
0

Natural Numbers
19
$\sqrt{4}$

Irrational Numbers
$\sqrt{2}$
1.121121112...
π
$-\sqrt{3}$

The number 0.24758326... is irrational because the decimal expansion is nonrepeating and nonterminating.

Try It!

Classify each number as rational or irrational.

π	3.565565556...
0.04053661...	-17
0.76	3.275

Convince Me! Construct Arguments Jen classifies the number 4.567 as irrational because it does not repeat. Is Jen correct? Explain.

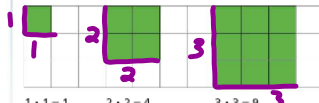
Rational	Irrational

EXAMPLE 2 Identify Square Roots as Irrational Numbers

Classify $\sqrt{3}$.

$\sqrt{3}$ means "the nonnegative square root of 3."

The **square root** of a number is a number that when multiplied by itself equals the original number. The radical symbol $\sqrt{\quad}$ is used to denote the nonnegative square root.



$1 \cdot 1 = 1$ $2 \cdot 2 = 4$ $3 \cdot 3 = 9$
 $\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$

The number 3 is not a perfect square, so $\sqrt{3}$ cannot be written as an integer. So, $\sqrt{3}$ is irrational.

A **perfect square** is a number that is the square of an integer. The first three integer perfect squares are 1, 4, and 9.

Generalize For any whole number b that is not a perfect square, \sqrt{b} is irrational.

EXAMPLE 3 Classify Numbers as Rational or Irrational

Classify each number as rational or irrational. Explain how you classified each number.

$-81,572$ $\sqrt{11}$ $5.636336333\dots$ $\sqrt{16}$

$-81,572$ is an integer and can be written as the fraction $-\frac{81,572}{1}$, so it is rational.

Rational	Irrational
$-81,572$	$\sqrt{11}$
$\sqrt{16}$	$5.636336333\dots$

11 is not a perfect square, so $\sqrt{11}$ is irrational.

The number 16 is a perfect square, so $\sqrt{16} = 4$ is rational.

This decimal expansion does not repeat or terminate, so it is irrational.


Try It!

Classify each number as rational or irrational and explain.

$\frac{2}{3}$ $\sqrt{25}$ -0.75 $\sqrt{2}$ $7,548,123$

Name: _____ Date: _____

n	Squared ⁿ n ²	Square Root \sqrt{n}	n	Squared ⁿ n ²	Square Root \sqrt{n}	n	Square Root \sqrt{n}	n	Square Root \sqrt{n}	n	Square Root \sqrt{n}
1	1	1.000	51	2601	7.141	101	10.050	151	12.288	201	14.177
2	4	1.414	52	2704	7.211	102	10.100	152	12.328	202	14.213
3	9	1.732	53	2809	7.280	103	10.149	153	12.369	203	14.248
4	16	2.000	54	2916	7.348	104	10.198	154	12.410	204	14.283
5	25	2.236	55	3025	7.416	105	10.247	155	12.450	205	14.318
6	36	2.449	56	3136	7.483	106	10.296	156	12.490	206	14.353
7	49	2.646	57	3249	7.550	107	10.344	157	12.530	207	14.387
8	64	2.828	58	3364	7.616	108	10.392	158	12.570	208	14.422
9	81	3.000	59	3481	7.681	109	10.440	159	12.610	209	14.457
10	100	3.162	60	3600	7.746	110	10.488	160	12.649	210	14.491
11	121	3.317	61	3721	7.810	111	10.536	161	12.689	211	14.525
12	144	3.464	62	3844	7.874	112	10.583	162	12.728	212	14.560
13	169	3.606	63	3969	7.937	113	10.630	163	12.767	213	14.595
14	196	3.742	64	4096	8.000	114	10.677	164	12.806	214	14.629
15	225	3.873	65	4225	8.062	115	10.724	165	12.845	215	14.663
16	256	4.000	66	4356	8.124	116	10.770	166	12.884	216	14.697
17	289	4.123	67	4489	8.185	117	10.817	167	12.923	217	14.731
18	324	4.243	68	4624	8.246	118	10.863	168	12.961	218	14.765
19	361	4.359	69	4761	8.307	119	10.909	169	13.000	219	14.799
20	400	4.472	70	4900	8.367	120	10.954	170	13.038	220	14.832
21	441	4.583	71	5041	8.426	121	11.000	171	13.077	221	14.865
22	484	4.690	72	5184	8.485	122	11.045	172	13.115	222	14.900
23	529	4.796	73	5329	8.544	123	11.091	173	13.153	223	14.933
24	576	4.899	74	5476	8.602	124	11.136	174	13.191	224	14.967
25	625	5.000	75	5625	8.660	125	11.180	175	13.229	225	15.000
26	676	5.099	76	5776	8.718	126	11.225	176	13.266	226	15.033
27	729	5.196	77	5929	8.775	127	11.269	177	13.304	227	15.067
28	784	5.292	78	6084	8.832	128	11.314	178	13.342	228	15.100
29	841	5.385	79	6241	8.888	129	11.358	179	13.379	229	15.133
30	900	5.477	80	6400	8.944	130	11.402	180	13.416	230	15.166
31	961	5.568	81	6561	9.000	131	11.446	181	13.454	231	15.199
32	1024	5.657	82	6724	9.055	132	11.489	182	13.491	232	15.232
33	1089	5.745	83	6889	9.110	133	11.533	183	13.528	233	15.264
34	1156	5.831	84	7056	9.165	134	11.576	184	13.565	234	15.297
35	1225	5.916	85	7225	9.220	135	11.619	185	13.601	235	15.330
36	1296	6.000	86	7396	9.274	136	11.662	186	13.638	236	15.362
37	1369	6.083	87	7569	9.327	137	11.705	187	13.675	237	15.395
38	1444	6.164	88	7744	9.381	138	11.747	188	13.711	238	15.427
39	1521	6.245	89	7921	9.434	139	11.790	189	13.748	239	15.460
40	1600	6.325	90	8100	9.487	140	11.832	190	13.784	240	15.492
41	1681	6.403	91	8281	9.539	141	11.874	191	13.820	241	15.524
42	1764	6.481	92	8464	9.592	142	11.916	192	13.856	242	15.556
43	1849	6.557	93	8649	9.644	143	11.958	193	13.892	243	15.588
44	1936	6.633	94	8836	9.695	144	12.000	194	13.928	244	15.620
45	2025	6.708	95	9025	9.747	145	12.042	195	13.964	245	15.652
46	2116	6.782	96	9216	9.798	146	12.083	196	14.000	246	15.684
47	2209	6.856	97	9409	9.849	147	12.124	197	14.036	247	15.716
48	2304	6.928	98	9604	9.899	148	12.166	198	14.071	248	15.748
49	2401	7.000	99	9801	9.950	149	12.207	199	14.107	249	15.780
50	2500	7.071	100	10000	10.000	150	12.247	200	14.142	250	15.811

KEY CONCEPT  can be written as a fraction

Numbers that are not rational are called irrational numbers.

Real Numbers

Rational Numbers $-\frac{4}{5}$ 0.75 31.8 $31\frac{8}{10}$	Integers -5 $-\frac{16}{4}$ $-1,000$ -4	Whole Numbers 0	Natural Numbers 19 $1, 2, 3, \dots$	Irrational Numbers $1.121121112...$ π $\sqrt{3}$
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The number 4 is a perfect square, so $\sqrt{4} = 2$ is rational.

The number 2 is not a perfect square, so $\sqrt{2}$ is irrational.

cannot write as a fraction;

Do You Understand?

- Essential Question** How is an irrational number different from a rational number?
- Reasoning** How can you tell whether a square root of a whole number is rational or irrational?
- Construct Arguments** Could a number ever be both rational and irrational? Explain.

Do You Know How?

- Is the number 65.4349224... rational or irrational? Explain.
- Is the number $\sqrt{2,500}$ rational or irrational? Explain.
- Classify each number as rational or irrational.
 4.27 0.375 $0.232342345...$ $\sqrt{6}$ $\frac{13}{1}$

Rational	Irrational
	$\sqrt{62}$

Name: _____

Practice & Problem Solving  

- Is 5.787787778... a rational or irrational number? Explain.
- Is $\sqrt{42}$ rational or irrational? Explain.

- A teacher places six cards, lettered A–G, on a table. Which cards show irrational numbers?



- Circle the irrational number in the list below.
 7.27 $\frac{5}{9}$ $\sqrt{15}$ $\sqrt{196}$
- Lisa writes the following list of numbers.
 $5.737737773...$, 26 , $\sqrt{45}$, $-\frac{3}{2}$, 0 , 9
 - Which numbers are rational?
 - Which numbers are irrational?

- Construct Arguments** Deena says that 9.565565556... is a rational number because it has a repeating pattern. Do you agree? Explain.

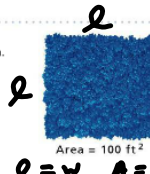
13. Is $\sqrt{1,815}$ rational? Explain.

14. Is the decimal form of $\frac{13}{3}$ a rational number? Explain.

15. Write the side length of the square rug as a square root. Is the side length a rational or irrational number? Explain.

$$\begin{aligned} \ell \cdot \ell &= 100 \text{ ft}^2 \\ \sqrt{\ell^2} &= \sqrt{100 \text{ ft}^2} \\ \ell &= 10 \text{ ft} \end{aligned}$$

The length of the rug is $\sqrt{100}$ ft. This is a rational number since $\sqrt{100} = 10 = \frac{10}{1}$ so it can be written as a fraction.



Area = 100 ft²
 $\ell = w$ $A = \ell \cdot w$

16. Reasoning The numbers 2.888... and 2.999... are both rational numbers. What is an irrational number that is between the two rational numbers?

17. Higher Order Thinking You are given the expressions $\sqrt{76 + n}$ and $\sqrt{2n + 26}$. What is the smallest value of n that will make each number rational?

Assessment Practice

18. Which numbers are rational?

- I. 1.1111111...
- II. 1.567
- III. 1.101101110...

- A. II and III
- B. III only
- C. II only
- D. I and II
- E. I only
- F. None of the above

19. Classify the following numbers as rational or irrational.

- $\frac{8}{5}$ π 0 $\sqrt{1}$ $4.46466...$ -6 $\sqrt{2}$

Rational	Irrational
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<input type="text"/>	<input type="text"/>
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