

TOPIC

# 2

## ANALYZE AND USE PROPORTIONAL RELATIONSHIPS

? Topic Essential Question


How can you recognize and represent proportional relationships and use them to solve problems?

Topic Overview

- 2-1 Connect Ratios, Rates, and Unit Rates
- 2-2 Determine Unit Rates with Ratios of Fractions
- 2-3 Understand Proportional Relationships: Equivalent Ratios
- 2-4 Describe Proportional Relationships: Constant of Proportionality
- 3-Act Mathematical Modeling: Mixin' It Up
- 2-5 Graph Proportional Relationships
- 2-6 Apply Proportional Reasoning to Solve Problems

Topic Vocabulary

- + constant of proportionality
- + proportion
- + proportional relationship




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Lesson Digital Resources

- INTERACTIVE ANIMATION** Interact with visual learning animations.
- ACTIVITY** Use with *Solve & Discuss It*, *Explore It*, and *Explain It* activities, and to explore Examples.
- VIDEOS** Watch clips to support 3-Act Mathematical Modeling Lessons and STEM Projects.
- PRACTICE** Practice what you've learned.

80
Topic 2 Analyze and Use Proportional Relationships


3-ACT MATH



# Mixin' It Up

**Mixin' It Up**

Drinking plenty of water each day is important. Water is necessary for everything your body does. Not drinking enough water can lead to health problems. It's even easier to drink enough water if you like the taste. There are many ways to make water more exciting. You can drink seltzer or filtered water. You can add fruit, vegetables, herbs, or flavor enhancers. You can add more or less based on what you like. Think about this during the 3-Act Mathematical Modeling lesson.



TUTORIALS

Get help from *Virtual Nerd*, right when you need it.

KEY CONCEPT

Review important lesson content.

GLOSSARY

Read and listen to English/Spanish definitions.

ASSESSMENT

Show what you've learned.

Additional Digital Resources

MATH TOOLS

Explore math with digital tools.

GAMES

Play Math Games to help you learn.

ETEXT

Interact with your Student's Edition online.

Topic 2 Analyze and Use Proportional Relationships
81

**TOPIC 2 STEM Project**

### Did You Know?

There are more than 326 million trillion gallons of water on Earth.

Only a small percentage of all this water is fresh water...

...and much of that fresh water is locked up in ice caps and glaciers.

**71% Water**

**29% Land**

The United Nations says that each person needs about 50 liters of water each day.

Each person in Africa has access to less than 20 liters of water each day.

Many doctors recommend that each person drinks eight 8-ounce glasses of water each day.

Each person in the United States uses on average about 35-40 gallons of water each day.

### Your Task: An Essential Resource

Access to fresh, clean water is important for human survival. You and your classmates will determine how much fresh water is available on Earth for people to use. You will also explore ways in which people have developed access to clean water.

82 Topic 2 STEM Project

**GET READY! TOPIC 2**

### Review What You Know!

**Vocabulary**  
Choose the best term from the box to complete each definition.

- The quantities  $x$  and  $y$  in the ratio  $\frac{x}{y}$  are called \_\_\_\_\_.
- $\frac{2 \text{ dogs}}{3 \text{ cats}}$  and  $\frac{10 \text{ dogs}}{15 \text{ cats}}$  are an example of \_\_\_\_\_.
- A(n) \_\_\_\_\_ is a type of ratio that has both terms expressed in different units.
- A(n) \_\_\_\_\_ has a fraction in its numerator, denominator, or both.

complex fraction  
equivalent ratios  
rate  
ratio  
terms

**Equivalent Ratios**  
Complete each equivalent ratio.

- $\frac{4 \text{ boys}}{7 \text{ girls}} = \frac{8 \text{ boys}}{\square \text{ girls}}$
- $\frac{16 \text{ tires}}{4 \text{ cars}} = \frac{\square \text{ tires}}{1 \text{ car}}$
- $\frac{8 \text{ correct}}{10 \text{ total}} = \frac{\square \text{ correct}}{50 \text{ total}}$
- $\frac{16 \text{ pearls}}{20 \text{ opals}} = \frac{8 \text{ pearls}}{\square \text{ opals}}$
- $\frac{32 \text{ pencils}}{8 \text{ erasers}} = \frac{8 \text{ pencils}}{\square \text{ erasers}}$
- $\frac{7 \text{ balls}}{9 \text{ bats}} = \frac{\square \text{ balls}}{27 \text{ bats}}$

**Rates**  
Write each situation as a rate.

- John travels 150 miles in 3 hours.
- Cameron ate 5 apples in 2 days.

**Equations**  
Write an equation that represents the pattern in the table.

x	4	5	6	7	8
y	12	15	18	21	24

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Topic 2 Analyze and Use Proportional Relationships 83

### Build Vocabulary

Use the graphic organizer to help you understand new vocabulary terms.

Definition	Key Characteristics
Examples	Non-Examples

**Proportion**

Definition	Key Characteristics
Examples	Non-Examples

**Constant of Proportionality**

Definition	Key Characteristics
Examples	Non-Examples

**Proportional Relationship**

### Explain It!

In a basketball contest, Elizabeth made 9 out of 25 free throw attempts. Alex made 8 out of 20 free throw attempts. Janie said that Elizabeth had a better free-throw record because she made more free throws than Alex.

ELIZABETH	ALEX
MADE	MADE
9	8
ATTEMPTED	ATTEMPTED
25	20



### Lesson 2-1 Connect Ratios, Rates, and Unit Rates

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**I can...**  
use ratio concepts and reasoning to solve multi-step problems.

**A. Critique Reasoning** Do you agree with Janie's reasoning? Explain.

**B. Construct Arguments** Decide who had the better free-throw record. Justify your reasoning using mathematical arguments.

### Focus on math practices

**Construct Arguments** What mathematical model did you use to justify your reasoning? Are there other models you could use to represent the situation?

**Essential Question** How are ratios, rates, and unit rates used to solve problems?

**EXAMPLE 1** Find Unit Rates

Nathan and Dan were both hired as lifeguards for the summer. They receive their paychecks for the first week. Who earns more per hour?

**Make Sense and Persevere**  
You can use a ratio to relate the number of hours worked and the amount earned.

LIFEGUARD SERVICES INC. EARNINGS STATEMENT	
EMPLOYEE	Dan Jones
HOURS	9
TOTAL EARNINGS	\$78.75

LIFEGUARD SERVICES INC. EARNINGS STATEMENT	
EMPLOYEE	Nathan Smith
HOURS	5
TOTAL EARNINGS	\$46.25

Draw a model to show how the quantities are related.



Find unit rates to determine how much each lifeguard earns each hour.

$$\frac{46.25}{5} = \frac{9.25}{1}$$

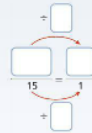
$$\frac{78.75}{9} = \frac{8.75}{1}$$

Nathan earns 50¢ more per hour.

**Try It!**

Jennifer is a lifeguard at the same pool. She earns \$137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns \$  per hour.

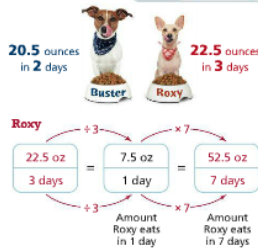
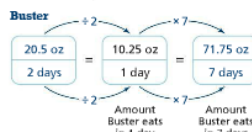


**Convince Me!** What do you notice about the models used to find how much each lifeguard earns per hour?

**EXAMPLE 2** Use Unit Rates

Brian agrees to watch his neighbor's dogs for 7 days. His neighbor provided a 128-ounce bag of dog food. Does Brian have enough food to feed the dogs all 7 days? Explain.

**STEP 1** Use unit rates to find how much each dog eats in 7 days.

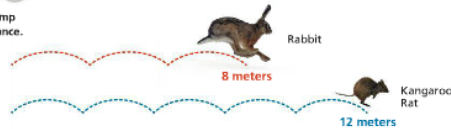


**STEP 2** Find the total amount of dog food needed for 7 days. Then compare.

$$71.75 + 52.5 = 124.25 \text{ and } 124.25 < 128, \text{ so Brian has enough dog food.}$$

**EXAMPLE 3** Compare Using Rates

Suppose that each jump covers the same distance. How many jumps does it take each animal to cover the same distance?



Make tables of equivalent ratios until the distance jumped is the same.

Rabbit		Kangaroo Rat	
Jumps	Meters	Jumps	Meters
3	8	5	12
6	16	10	24

× 3      × 2

The rabbit jumps 24 meters in 9 jumps.  
The kangaroo rat jumps 24 meters in 10 jumps.

**Try It!**

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds. A bathroom sink faucet streams 0.75 gallon of water in 18 seconds. Which faucet will fill a 3-gallon container faster?

**KEY CONCEPT**

You can use equivalent ratios and rates, including unit rates, to compare ratios and to solve problems.

$\frac{45.5}{7} = \frac{?}{1}$

$\frac{45.5}{7} = \frac{6.5}{1}$

**Do You Understand?**

- Use Structure** How are ratios, rates, and unit rates used to solve problems?
- Use Structure** Dorian buys 2 pounds of almonds for \$21.98 and 3 pounds of dried apricots for \$10.99. Which is less expensive per pound? How much less expensive? Complete the tables of equivalent ratios to help you solve.
- Generalize** How are unit rates and equivalent ratios related?

Almonds		Dried Apricots	
Cost	Weight (lb)	Cost	Weight (lb)
\$21.98	2	\$10.99	3

**Do You Know How?**

- Krystal is comparing two internet service plans. Plan 1 costs \$34.99 per month. Plan 2 costs \$134.97 every 3 months. If Krystal plans to stay with one service plan for 1 year, which should she choose? How much will she save?
- Pam read 126 pages of her summer reading book in 3 hours. Zack read 180 pages of his summer reading book in 4 hours. If they continue to read at the same speeds, will they both finish the book after 5 total hours of reading?
- Nora and Eli are making homemade spring rolls for a party. Nora can make 8 spring rolls in 10 minutes. Eli can make 10 spring rolls in 12 minutes. If they each make 40 spring rolls, who will finish first?

10.99  
- 8.75  
-----  
2.24

The apricots are \$2.24 cheaper per pound than the almonds.

"per" or "each" } unit rate

**Pam**

$$\frac{126 \text{ pg}}{3 \text{ hr}} \div 3 = \frac{42 \text{ pg}}{1 \text{ hr}}$$

**Zack**

$$\frac{180 \text{ pg}}{4 \text{ hr}} \div 4 = \frac{45 \text{ pg}}{1 \text{ hr}}$$

**Total pg**

$$\begin{array}{r} 42 \\ \times 5 \\ \hline 210 \end{array}$$

**Total pg**

$$\begin{array}{r} 45 \\ \times 5 \\ \hline 225 \end{array}$$

**4 hr**  $\div 4 = 1 \text{ hr}$

**4 hr**  $\div 4 = 1 \text{ hr}$

Name: \_\_\_\_\_

**Practice & Problem Solving**

**Leveled Practice** In 7-8, complete the tables of equivalent ratios to solve.

7. After Megan walked 5 miles, her activity tracker had counted 9,780 steps. David's activity tracker had counted 11,928 steps after he walked 6 miles. Suppose each person's step covers about the same distance. Who takes more steps to walk 1 mile? How many more steps?

Megan's Steps		David's Steps	
Steps	Miles	Steps	Miles
9,780	5	11,928	6
1,956	1	1,988	1

David takes more steps to walk 1 mile. 1,956 more steps for 1 mile

8. A package of 5 pairs of insulated gloves costs \$29.45. What is the cost of a single pair of gloves?

Price	Pairs of Gloves
	5
	1

One pair of gloves costs \_\_\_\_\_.

9. Which package has the lowest cost per ounce of rice?

White Rice  
12 ounces  
\$4.50

White Rice  
18 ounces  
\$6.30

10. A nursery owner buys 5 panes of glass to fix some damage to her greenhouse. The 5 panes cost \$14.25. Unfortunately, she breaks 2 more panes while repairing the damage. What is the cost of another 2 panes of glass?

11. **Be Precise** An arts academy requires there to be 1 tutor for every 6 students. How many tutors does the academy need if it has 120 students?

12. **Make Sense and Persevere** In large cities, people often take taxis to get from one place to another. What is the cost per mile of a taxi ride? How much is a 47-mile taxi ride?

$$\begin{array}{r} 1988 \\ -1956 \\ \hline 32 \end{array}$$

extra info (not needed)

$$\frac{72 \text{ stu} \div 6}{6 \text{ tut} \div 6} = \frac{12 \text{ stu}}{1 \text{ tut}}$$

$$\frac{12 \text{ stu} \times 10}{1 \text{ tut} \times 10} = \frac{120 \text{ stu}}{? \text{ tut}}$$

The arts academy needs 10 tutors for the 120 students.



13. The track team needs new uniforms. The students plan to sell plain toy tigers (the school mascot) for \$5. The students find three companies online that sell stuffed mascots.

a. Which company has the lowest cost per tiger?

b. If they use that company, how much profit will the students make for each tiger sold?

14. A contractor purchases 7 dozen pairs of padded work gloves for \$103.32. He incorrectly calculates the unit price at \$4.76 per pair.

a. What is the correct unit price?


b. **Critique Reasoning** What error did the contractor likely make?

15. **Higher Order Thinking** A warehouse store sells 5.5-ounce cans of tuna in packages of 6. A package of 6 cans costs \$9.24. The store also sells 6.5-ounce cans of the same tuna in packages of 3 cans for \$4.68. It also sells 3.5-ounce cans in packages of 4 cans for \$4.48. Which package has the lowest cost per ounce of tuna?

Company A  
12 tigers for \$33.24

Company B  
18 tigers for \$44.80

Company C  
15 tigers for \$41.10



**Assessment Practice**

16. Lena is making two dishes for an event. Each batch of her mac-and-cheese recipe calls for 6 ounces of cheese and 2 tablespoons of basil. For every two pizzas, she needs 16 ounces of cheese and 5 tablespoons of basil.

**PART A**  
Lena buys a 32-oz package of cheese. Does she have enough cheese to make 2 batches of mac-and-cheese and 3 pizzas? Explain.

**PART B**  
Lena decides to make 1 batch of mac-and-cheese and 3 pizzas. How many tablespoons of basil does she need? Explain your answer.

17. Irene's car had 6 gallons of gas in its 15-gallon tank. Irene wants to fill it at least half way. If gas costs \$4.70 per gallon, which of the following statements is true? Select all that apply.

Irene needs to add at least 1.5 gallons of gas to the car's gas tank.

If Irene adds \$3.80 worth of gas, her tank will be more than half full.

If Irene adds \$7.60 worth of gas, her tank will be more than half full.

If Irene filled her tank, it would cost \$34.20.

needs to pay at least

Currently has 6gal

Total capacity 15gal

she wants at least

$$\frac{1}{2} = \frac{1}{2}(15) = 7.5 \text{ gal}$$

$$\begin{array}{r} 7.5 \\ -6.0 \\ \hline 1.5 \end{array} \text{ (needs to fill at least)}$$

$$15 - 6 = 9$$

$$\begin{array}{r} 7.5 \\ \times 3.80 \\ \hline 28.50 \\ 30.60 \\ \hline 28.50 \\ 30.60 \\ \hline 34.20 \end{array}$$
