

MATHCOUNTS®



11TH ANNUAL MATHLETES IN ACTION NOVEMBER 8, 2014

SPRINT ROUND
PROBLEMS 1-25

NAME _____

SCHOOL _____

DO NOT BEGIN UNTIL INSTRUCTED TO DO SO.

This round of the competition consists of twenty-five problems. You will have 32 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wristwatch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the designated space on the problem booklet. If you complete the problems before time is called, use the remaining time to check your answers.

TOTAL CORRECT	SCORER'S INITIALS

1. Elina has twelve stamps, some of which are \$ 0.33 stamps and some of which are \$ 0.23 stamps. If the total value of all the stamps is \$3.16, how many of her stamps are worth \$0.33? 1. _____ stamps

2. A 2-cup mixture consists of $\frac{2}{3}$ cup of flour and the rest is nuts. If 1 cup of flour is added to make a 3-cup mixture, what fraction of the 3-cup mixture is flour? Express your answer as a common fraction. 2. _____

3. A rectangular field is to be fenced with posts placed at each corner and along each side, evenly distributed so that the centers of the posts are 6 feet apart along each side. If the field is 60 yards long and 80 yards wide, how many posts are needed to build such a fence? 3. _____ posts

4. Suppose the radius of a cone is doubled and the volume remains the same. Express as a fraction the relationship of the new height to the original height? 4. _____

5. How many six-inch by six-inch square tiles are needed to cover a three-foot by two-foot rectangular floor? 5. _____ tiles

6. The San Antonio Knights little soccer team is comprised of 6 boys and 12 girls. If two different team mates are randomly selected from the team members, what is the probability that both are girls? Express your answer as a common fraction. 6. _____

$$\frac{2}{3} \cdot \frac{11}{17} = \frac{22}{51}$$

7. If one liter is 1000 cubic centimeters, how many cubic centimeters are in 4.2 liters?

7. _____ cubic cm

8. The sum of three positive integers is 20, and the smaller two integers differ by 5. What is the smallest possible value of the largest integer?

8. _____

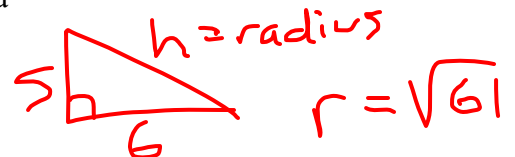
9. If the chord of a circle is 12 feet long, and the distance from the chord to the circle's center is 5 feet, then what is the area of the circle? Express your answer in terms of π .

9. _____ square feet



$$5^2 + 6^2 = r^2$$

$$25 + 36 = r^2$$



10. How many hours will a car traveling at 45 miles per hour take to catch up with a car traveling at 30 miles per hour if the slower car starts one hour before the faster car?

10. _____ hours



$$A = \pi r^2$$

$$(\sqrt{61})^2 = 61\pi$$

11. If $y = x^2 + 10x + 21$, what is the least possible value of y?

11. _____

(A) (B) (C)

$$A \text{ of } \underset{(x)}{5} = \frac{-b}{2A} = \frac{-10}{2(1)} = \underline{-5}$$

$$y = (-5)^2 + 10(-5) + 21$$

$$25 + -50 + 21$$

$$+46 + -50$$

12. Twenty-seven increased by twice a number is 39. What is the number?

12. _____ (-4)

$$27 + 2n = 39$$

$$2n = 12$$

$$n = 6$$

\$38.50
each Sat.

13. Dawn estimates she will need \$750 for the computer she wants. She baby-sits for the Kidd family every Saturday for \$7.00 per hour. On average, she baby-sits 5.5 hours each Saturday. What is the fewest number of weeks she must work in order to save enough money for the computer she wants?

$$38.5 \sqrt{7500}$$

$$\begin{array}{r} 110 \\ 7 \overline{) 770} \\ \underline{77} \\ 0 \end{array}$$

$$13. \frac{20}{107} \text{ weeks} \rightarrow 108 \text{ hrs}$$

$$7 \overline{) 750}$$

$$\begin{array}{r} 107 \\ \underline{7} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

$$11 \overline{) 108}$$

$$\begin{array}{r} 9 \\ \underline{99} \\ 9 \end{array}$$

14. A right circular cylinder has a volume of $144\pi \text{ cm}^3$ and a height of 9 cm. What is the radius of the base?

$$V = \pi r^2 h$$

$$144 = \pi r^2 9$$

$$\sqrt{16} = \sqrt{r^2}$$

$$9 \overline{) 144}$$

$$\begin{array}{r} 16 \\ \underline{9} \\ 54 \\ \underline{54} \\ 0 \end{array}$$

14. 4 cm

15. What is the largest prime factor of 78?



15. 13

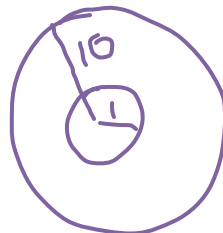
16. Track practice lasts for one hour from 2:30 – 3:30. At a randomly selected time during track practice, Emma looks at her watch. What is the probability that the minute and hour hand on her watch form an acute angle? Express your answer as a common fraction.

16. _____

17. Carol, Jane, Kim, Nancy, and Vicky competed in a 400-meter race. Nancy beat Jane by 6 seconds. Carol finished 11 seconds behind Vicky. Nancy finished 2 seconds ahead of Kim, but 3 seconds behind Vicky. How many seconds did Kim finish ahead of Carol?

17. _____ seconds

18. Two concentric circular regions have radii of 1 unit and 10 units. What is the area, in square units, outside the smaller region and inside the larger region? Express your answer in terms of π .



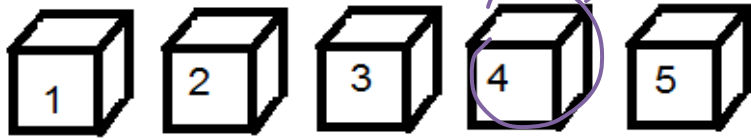
$$A_o = \pi r^2 = \pi (10)^2 = 100\pi$$

$$A_o = \pi (1)^2 = 1\pi$$

$$\frac{99\pi}{99\pi}$$

18. _____ sq units

19. Hugh has five boxes labeled 1,2,3,4 and 5 which are arranged in increasing order from left to right. He wants to get them into descending order from left to right. To do this, he will repeatedly switch the order of two adjacent boxes. What is the fewest number of switches needed to achieve the desired order? 19. _____



20. Bobby and Sarah drove the same distance. Sarah drove 20% faster than Bobby and she arrived half an hour earlier. How many hours did Bobby drive? 20. _____ hours

$$d = r \cdot t$$

21. What is the ones digit when 9^{36} is expressed as an integer? 21. _____

$$\begin{aligned} 9^0 &= 1 \\ 9^1 &= 9 \\ 9^2 &= 81 \\ 9^3 &= 243 \\ 9^4 &= 1 \end{aligned}$$

22. The first three terms of an arithmetic sequence are p , 6 and $2p-3$. What is the tenth term of this sequence? 22. _____

23. A bag contains black and silver balls. If 5 balls will be pulled from the bag with replacement, the probability of getting exactly 3 black balls is 32 times the probability of getting exactly one black ball. What percent of the balls originally in the bag are black? 23. _____ %

$$E = \left(\frac{1}{2}A\right) \quad E + 4 = \frac{2}{3}(A + 4)$$

24. Emaleigh's age is half of Addie's age. In four years, Emaleigh's age will be two-thirds of Addie's age. How old is Emaleigh now? 24. _____ years

25. Darla can have pennies, nickels, dimes and quarters in her purse. The average value of all coins in her purse is 20 cents. If she adds one more quarter to her purse, the average value of all the coins will be 21 cents. How many nickels does she have in her purse? 25. _____ nickels